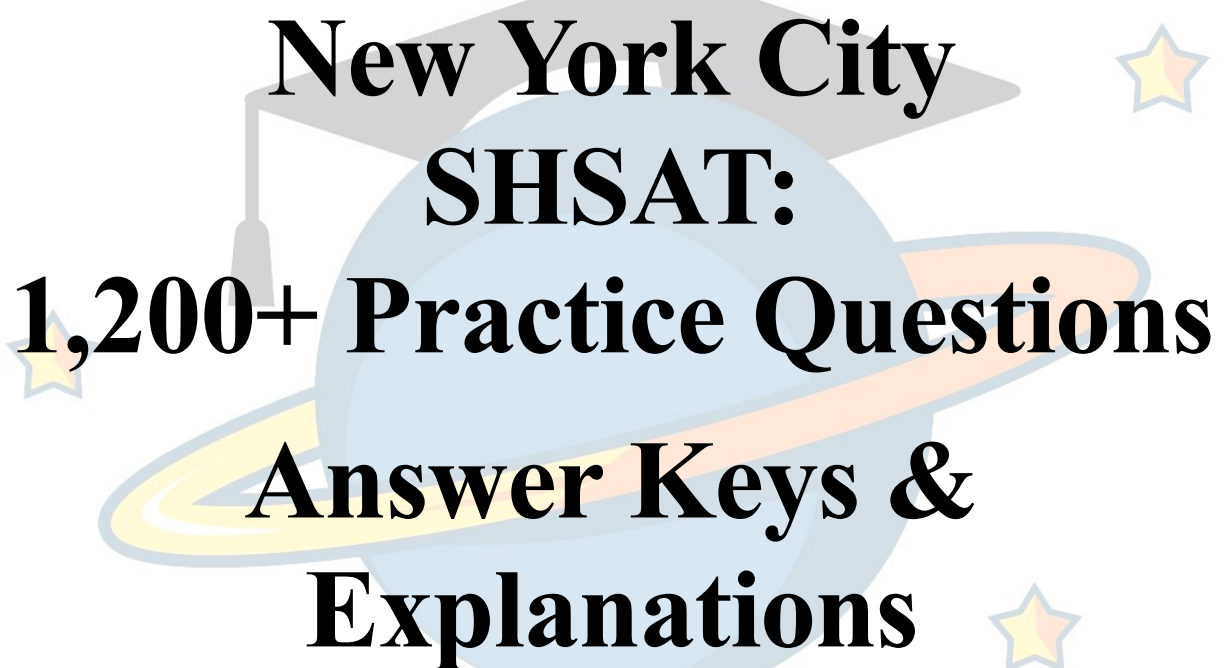


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# **New York City SHSAT: 1,200+ Practice Questions Answer Keys & Explanations**

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Note: This answer key and the accompanying explanations apply to the 2024 edition of the workbook, copyrighted in 2024. If you are looking for answer key and explanations for the 2023 edition (copyrighted in 2023), please visit us at [www.thetutorverse.com/books](http://www.thetutorverse.com/books) to submit a [help ticket request](#). Please note that we are not able to support answer keys and explanations for earlier editions of the workbook.

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## Answer Explanations

This section provides answer solutions and explanations to the practice tests, questions, and sections of the workbook.

### Diagnostic Practice Test (Form A)

#### Part One: English Language Arts

1. *C. Modifiers.* Modifiers are words or phrases that describe (or modify) other parts of a sentence. Modifiers tell us more information and paint a more detailed picture about a sentence's primary nouns and verbs. Modifiers can be adjectives, adverbs, adjective or adverb clauses, and verb or prepositional phrases. In general, modifiers should be placed as closely as possible to the thing that it modifies. Confusion can arise when the modifiers are placed "far away" from the things they modify. In this case, the modifier is "with a bushy wagging tail," and it is misplaced. It doesn't make sense for this to modify the "soccer ball." Instead, it should modify the dog. This means we should put it close to "dog," and the only place that suits this is after "dog" and before "chased." The other arrangements confuse the meaning of the sentence. In some, it can be construed that the dog actually chased after the ball along with a bushy wagging tail (a strange image!).
2. *F. Logical Comparison.* The noun being compared is "temperature." In this case, we are comparing temperatures in Hawaii with temperatures in Antarctica. As written, the sentence compares the temperatures in Hawaii with Antarctica itself. This is the same for the other answer choices. Only by using "those in" is it clear that the comparisons being made are between the temperatures of two different locations (as opposed to the temperature of one location with another location itself).
3. *A. Possessive Nouns & Determiners.* We can split the first sentence into two smaller sentences: "Since chili was my sister favorite food, we decided to enter the chili contest at the county fair" and "Since chili was my favorite food, we decided to enter the chili contest at the county fair." Notice that "sister" is missing an "s". We need the comma after "contest" and "food" because they are at the end of a subordinate clause (the respective sentences start with "since" and "during"). Therefore, we can eliminate B, C, and D. "Ours" is the appropriate way to signal possession of the collective "our".
4. *F. Organization & Logic.* The sentence in question states that King made references to "sources." Sources are listed in sentence 6, which also refers to "these." Without having "sources" immediately preceding it, we would not know that "these" refers to "sources."
5. *C. Organization & Logic.* The previous sentence describes how King thought that these allusions would have made the audience like his speech more. Sentence 8 describes how this did not actually happen. Therefore, the best transition word to use should signify a contrast, as "nonetheless" does.
6. *E. Organization & Logic.* Sentence 9 describes how King did not follow his speech, and improvised. However, it does so in a roundabout way, including redundancies ("departed from his carefully written speech" and "not following his prepared speech"; "improvising" and "making it up as he went along"). Improvising is itself the act of coming up with a speech on the spot, and the first choice expresses this most clearly. We do not need to also say that King stopped following his prepared speech, since we already know that he "began to improvise."
7. *C. Organization & Logic.* Sentence 16 describes the speaking tactic called anaphora. Sentence 17 says that "this" (referring to anaphora) is "the repetition of a phrase at the beginning of sentences." Sentence 18 goes on to describe that for King, this was the phrase "I have a dream." Therefore, we know that sentence 17 provides more information about sentence 16, so we should not use a conjunction like "not" or even "and," since this suggests that anaphora and repetition are not the same thing. Neither should we use "because," since there is no causal relationship here. Instead, we need simply to connect the two sentences with a "which is" and a comma to describe more about "anaphora."

8. H. *Organization & Logic*. The paragraph describes specifically how King used various techniques to enhance his speech, making it memorable. The sentence states “these tactics,” so it must refer to specific tactics, and must come after the other sentences. This sentence also describes an effect, which is best described after the causes (i.e. the varied use of volume).
9. A. *Tenses, Mood, & Number*. Though it would be correct to say that “King moved the hearts and minds of an entire generation,” the sentence uses the phrase “was able,” which means we must use “move” in the present tense. This is because the phrase “to move...generation” is an infinitive phrase. The actual core of the sentence is “he was able.” “Thanks to...speech” is a dependent clause (which needs to be separated from the rest of the sentence with a comma) that describes more about what he was able to do, just as the infinitive phrase does. Changing “was” to “has been” does not change this fact. The word “an” signifies a single entity, so “generations,” which is plural, would be incorrect.
10. F. *Organization & Logic*. Sentence 11 describes some of King’s biographical information, which is not the focus of the passage and is irrelevant given the context of the rest of the paragraph. The passage is concerned with one of King’s speeches, and how it was effective. The other choices are all important aspects of this idea.

#### The Cold

11. D. *Main Idea*. Lines 1-2 (“In the city, the cold is / thin and vain”) and lines 31-21 (“In the country, the cold is / Soft. Soft”) contrast how different the cold feels in the two places. Though it introduces the cold in the city and the cold in the country as the main characters of the poem, the poem is not a dialogue between them.
12. F. *Craft & Structure*. The comparison of the cold to a blade, or saber, shows how it penetrates protections: “shearing through feather and wool and bone.” It is not a literal blade that disfigures people or shaves off their fur and feathers.
13. A. *Main Idea*. The city is known for being a busy, rushed place – attributes which the cold takes on – while the country is known for being lush and abundant. They show how the cold is different because it takes on the characteristics of its locale. Though later in the poem, it is revealed that the two meet, these two excerpts are not referring to them attempting to meet one another.
14. G. *Craft & Structure*. The descriptions of outer appearances of the cold of the city shows how sleek and sharp it is, as he wears a “suit of ice” and “granite topcoat.” On the other hand, the descriptions of the cold of the country are soft and cozy: “her fleecy white coat/...her magnificent hoary cloak.” These descriptions are metaphorical rather than literal, and not meant to show different degrees of cold.
15. D. *Craft & Structure*. This The stanza begins with “The buildings stand taller, sharper as he passes, / polished and menacing,” while the two quoted lines show how the city can be beautiful as well. The “diamond necklace” is a simile of something beautiful, not a literal present.
16. F. *Craft & Structure*. At the end of the poem, the cold in the city and the cold in the country are revealed to be romantically connected. These two phrases hint at this connection, implying that each one is searching for the other. They are not directly addressing the reader, despite the use of “you.”
17. A. *Craft & Structure*. The line “chill-y! chilly-chilly-chilly-chill-EE!” is a play on the word “chilly,” where the alternate spelling “chill-EE” mimics the literal sound of birdcalls, while the meaning of the word describes how cold it is. This supports an image of playfulness for the cold of the country, who is being contrasted as the opposite of the serious and sharp cold of the city throughout the poem. The emphasis of this moment is not on the birds suffering in winter.
18. G. *Craft & Structure*. The description, “before the wind can take a breath, / swoops through” shows how quickly the cold in the country moves, out of eagerness to embrace the city-cold lingering on the visitors. The poem does not characterize the country cold as “reckless” anywhere else, so it is unlikely that that is the main point here.



19. C. *Craft & Structure*. The poem establishes the longing between the two colds at earlier points (lines 12, 15, 51, 53-56), and this last stanza ends on a somber note, as it emphasizes how rarely the two entities are able to meet despite their feelings.

#### A Garden Masterpiece

20. F. *Main Idea*. In paragraphs 1-2, the passage discusses how Monet and his series of paintings Water Lilies are some of the most influential Impressionist paintings. As discussed in paragraph 5, Impressionism focuses on perception rather than appearance. Though Impressionist artists challenged the conventions of established art institutions of the time, there is no evidence that Monet personally hated them. The idea that Monet escaped poverty is not discussed in the passage, and his near-blindness is a supporting detail rather than the central idea.
21. C. *Supporting Idea*. The author discusses the classical and realistic art in studios in the likes of École des Beaux in order to contextualize the artistic choices that Impressionists made as innovators. There is no mention of them once studying at these academies, or of art schools in the post-Impressionist eras.
22. H. *Craft & Structure*. “Idealized” means representing the most perfect, or typical, example. By using the word to describe the type of landscapes that artists during that time painted, the author implies that the landscapes were quintessential versions of nature rather than reality. This is also supported by the detail that they painted landscapes in their studios, and not outside. This particular sentence does not discuss the realism of the landscapes.
23. D. *Inference*. Paragraph 3 explains that Monet painted the same scene over and over again in order to capture how it looked at different times of day; we can infer that the changes he was observing were in appearance, light, and shadows as the sun moved in the sky. He was not a perfectionist, but an impressionist, and the passage does not connect his poor vision to painting scenes repeatedly (in fact, it says that the poor vision led to powerful and beautiful works).
24. E. *Main Idea*. Since Monet wanted to capture his perception of his garden rather than its realistic appearance, the paragraph describes how Monet’s paintings embodied the artistic concepts of sensation that he valued. The passage discusses his renown for painting the series in paragraph 3, not 5. Similarly, Monet’s high regard for his garden is mentioned in paragraph 4.
25. B. *Supporting Idea*. Since Monet was able to portray his perceptions of the garden despite not being able to see the details accurately, Monet truly embodied the value of perception over accuracy. The large size of his paintings does not prove anything about his style, and having two compositions could just as well be realistic as subjective.
26. G. *Inference*. Throughout the passage, the author describes how Impressionists were concerned with perception and feeling as opposed to details and made-up ideals. This rules out all other answer choices. One can use process of elimination here to note that the other choices describe things that are the same, and therefore cannot all be the correct choice.
27. A. *Supporting Idea*. Paragraph 7 explains why Monet used different sized canvases, specifically large ones. Monet wanted to give the illusion of a wave with no end point, no horizon or shore. This would evoke a sensation of peace. The two different styles of compositions are distinct and Monet might have experimented with different canvases with them. However, they do not explain Monet’s reasoning behind choosing large canvases.

#### The Rule of Law

28. H. *Main Idea*. The passage shows the progression of rule of law from being a theory in ancient China and Greece, to becoming realized through the Magna Carta and American Constitution, and eventually popularized. Though rule of law is used in the Constitution, the idea was not invented by Americans.

29. A. *Inference*. The text states that, under the rule of law, “everyone is bound by the same rules and laws that govern society.” This shows an inherent equality in the law, whereby everyone is held equally accountable. Therefore, being a political leader does not make one exempt from codified law under any circumstances.
30. E. *Supporting Idea*. The paragraph highlights the stratocracies, monarchies, and plutocracies that existed prior to the rule of law’s general acceptance within the global community, though there is no mention of them being more or less successful. The next paragraph focuses on the birth of the rule of law in ancient society, but paragraph 3 focuses mainly on the governments that created power structures that were antithetical to the rule of law.
31. D. *Inference*. The author makes no comparisons between Aristotle and Han Fei, instead describing how they shared a similar idea. Rule by the strongest is a stratocracy, and rule by a single person is a monarchy; neither is a type of rule that the author thinks highly of. We only know for certain that the U.S. Constitution was heavily influenced by a similar document in medieval England, the Magna Carta.
32. H. *Supporting Idea*. The text implies that Han Fei and Aristotle had a high regard for the equalizing power of the rule of law. Conceptually, they believed nobody could rule above the law, so they would have frowned upon the consolidation and abuse of power described in options F and G.
33. D. *Main Idea*. According to the text, the signing of the Magna Carta is what allowed the theoretical concept of the rule of law to be implemented. The rule of law allowed for a kind of legal equality that was not reflected in monarchies or plutocracies.
34. E. *Inference*. Other nations tried to replicate, or imitate, the U.S. Constitution because it showed them that the rule of law was possible to implement, and lead to a successful and powerful society. These nations did not fear the Constitution or try to undermine it.

Prove It!

35. D. *Main Idea*. Although the first paragraph discusses how, in the past, “people used myths and fantastical tales to explain the world around them,” the rest of the passage describes how the scientific method has helped people better understand natural phenomena more accurately. Galileo’s experiment is only referenced as a detail to support this central idea. Additionally, while gravity (i.e., falling objects) is referenced as one kind of natural phenomenon that has been studied over time using the scientific method, the passage is about how perceptions of all-natural phenomena have changed over time, due to the scientific method.
36. G. *Craft & Structure*. The entire passage discusses how scientific knowledge has become more accurate and objective thanks to the modern employment of the scientific method. However, paragraph 1 sets the stage for this larger discussion by explaining how early understandings of science were simplistic and fanciful, with examples like earthquakes being caused by giant catfish. This purpose of this paragraph is neither to highlight the role of nature in inspiring myths, nor to defend the accuracy of these myths. Its sole purpose is to transition the text into a larger discussion on the utility of the scientific method. Likewise, paragraph 1 does not reference gravity like the other paragraphs.
37. A. *Craft & Structure*. The words “repeatable” and “observable” are used in paragraph 2 to argue that there is a certain dependability and validity that emerges from the scientific method due to its repetitious character and its objective intent. The paragraph does not imply that only scientists can use the scientific method.
38. F. *Craft & Structure*. The word “suspected” means “thought of with suspicion,” which implies that Galileo’s active skepticism, or thought process, was necessary to the development of his experiment. This is a part of the scientific method, where scientists observe and research, then come up with a claim about how the phenomenon works. Galileo was not being paranoid, as his suspicions were

then supported by evidence. It is implied that this reaction was different from the ideas of other scientists at the time, not similar.

39. B. *Main Idea*. The fifth paragraph describes Galileo's gravity experiment at the Leaning Tower of Pisa – a historical example of using the scientific method – and how it led Galileo “to conclude – rightly” an important scientific concept. There is no indication in the text that the scientific findings of the experimenter were not supported by later re-enactments.
40. E. *Supporting Idea*. Paragraph 6 discusses how air resistance causes objects to fall to the ground at different rates. In a vacuum, there is no air resistance; therefore, a feather and a hammer would fall at the same rate in a vacuum. A feather and hammer do not fall at different speeds due to their mass (which Galileo proves is not the cause behind different speeds of free fall) or the materials that they are made of.
41. C. *Main Idea*. Paragraph 7 discusses how modern-day scientists have reproduced Galileo's experiments in space, where they would be more accurate (due to air resistance). The last sentence, “Their results confirmed Galileo's findings – and demonstrate how the scientific method can illuminate truth,” shows that this information was included to show how Galileo's experiment is an example of the scientific method, due to this reproducibility.

Adapted from *The Scarlet Plague*

42. E. *Craft & Structure*. Following this phrase, Granser goes on to detail all the ways in which the world before the Scarlet Plague were vastly different from now (restaurants and airships, instead of bears). Given his audience are boys who had never seen that past world, we can infer that he is using a marveling tone. There is no evidence that he feels his loss is unjust, and his nostalgia is for the world that has changed, not his individual body.
43. D. *Craft & Structure*. Granser refers to things like restaurants, ships, and crowds of people, which are associated with present times, as being from the far past. Therefore, the setting of the story must be in a hypothetical future, after the Scarlet Plague. It is not a realistic fiction story, nor is there evidence that it takes place on another planet.
44. G. *Supporting Idea*. In this answer option, Edwin is demonstrating his confusion by asking about the meaning of a word which Granser uses freely. Option F is an example of the boys correcting each other's crude speech, but does not show that they cannot understand Granser's words.
45. B. *Craft & Structure*. “Garrulous” means overly talkative, or talking often of trivial and unimportant things. The boys are shown to ignore Granser, but not to be frustrated or annoyed by him. In fact, later they request one of his stories. Though talkative, Granser is never shown as irritable or grumpy.
46. H. *Supporting Idea*. Paragraph 4 offers the only detail throughout the story that shows the new world in a positive light as opposed to a negative one, in comparison to the past. However, Granser does not appear to be arguing that the current world is superior or fundamentally similar to the past, and seems to be saying it as a random aside.
47. B. *Inference*. The story poses a contrast throughout of Granser's educated thinking and the uneducated, more “primitive” nature of the boys. The scene with the wolves shows how much the boys fit in with this current, wilder world, and the Biblical reference to David and Goliath emphasizes this. The boys show no terror of the wolves, since they react to it by laughing and lying back in the sand. Also, the story itself of David and the Goliath does not describe David as being “confident in dealing with dangerous predators” (instead, he is the underdog), so that cannot be why the author included the simile.
48. E. *Craft & Structure*. The metaphor connecting human progress to foam supports the idea that all human labor eventually disappears: as foam dries out in the sand, so too did the markings of civilization disappear after the plague. The reference to ‘foam’ is not a literal one, and there is no mention in any other part of the story of sea levels rising or tidal patterns.



49. *C. Inference.* In paragraph 10, Hare-Lip speaks rudely to Granser. However, in paragraph 15, when the boys ask Granser a question, Granser exhibits great patience in teaching their number concepts and explaining the word. The boys tolerate Granser throughout the story, but are not particularly patient with him. While it is true that they have very different backgrounds, Granser does not argue constantly with the boys.

#### Helping Others Pays Off

50. *F. Craft & Structure.* Prior to this excerpt, the author states “it is hard to find a job that has all the desired qualities” (paragraph 1), and the two quoted sentences are examples to support this idea. As such, though the specific drawbacks (stressful versus not satisfying) and benefits (pays well) may be different, the parallel structure is emphasizing the imperfect nature of jobs. It is not discussing the nursing industry specifically, as that is introduced in paragraph 2.
51. *C. Main Idea.* Over the course of the passage, the author describes the various benefits of being an RN. She mentions in passing how to become an RN, as well as different specializations, but those are subtopics addressed by individual paragraphs and not the entire passage. Although drawbacks are mentioned in paragraph 4, paragraph 5 immediately follows up with a counterexample, and the tone of the whole passage is favorable to the nursing career.
52. *H. Supporting Idea.* The central idea of paragraph 2 is stated in its last sentence: “Because there are so many options for specialization, RNs can choose to work in different environments and roles that best suit their interests and strengths” (paragraph 2). The diverse settings that they work in, and their various duties, are details to support this idea.
53. *B. Main Idea.* In the third paragraph, the author discusses how the career has a positive outlook primarily because the world’s population “continues to grow” and “demand for healthcare professions” is also expected to grow. The author explicitly states that compensation is predicted to grow along with the number of open positions. There is no mention of staff turnover, or funding from the Department of Labor.
54. *G. Craft & Structure.* Paragraph 4 is the only one that lists negative aspects, or counterarguments to becoming an RN. Paragraph 5 then challenges this counterargument by showing how nurses are generally happy with their jobs despite the challenges. Paragraph 4 is not related to the ideas introduced in paragraph 3 (the growth of the job), or paragraph 6 (degrees necessary to become an RN).
55. *D. Supporting Idea.* The quotes in paragraph 5 are specific anecdotes for why individual nurses love their job, and follow the statistics describing overall job satisfaction in the industry. They are not particularly dramatic, and do not offer a new argument to support the overall claim.
56. *H. Supporting Idea.* The graph shows that the job growth from 2006-2016 for RNs has far exceeded those of other healthcare jobs. The chart does not provide information about length of age, compensation, or the length of illnesses.
57. *A. Inference.* The last sentence of the passage states: “Many find that these requirements are a small price to pay to be a part of such a well-respected and well-compensated profession” (paragraph 7), implying that the effort needed to get the necessary degrees for nursing are well worth it (in job satisfaction, career opportunities, and compensation). Given the information in paragraph 4, the passage does not argue that nursing is an occupation with zero drawbacks.

**Part Two: Math**

58. *–424. Numbers & Operations – Absolute Value.*  $|150 - 510| = |-360| = 360$ , and  $|15 - 51| = |-36| = 36$ , so the equation simplifies to  $36 - 360 - q = 100$ . Combine like terms, and isolate the variable for  $-q = 424$ , and finally  $q = -424$ .
59. *0.4. Algebra – Algebraic Expressions & Equations.* Simplify the equation by distributing to get rid of parentheses:  $9 - 2x - 10 = 3x - 5 + 5x$ . Then combine like terms:  $-1 - 2x = 8x - 5$ . Adding  $2x$  and  $5$  to both sides results in  $4 = 10x$ . Therefore,  $x = 4/10$ , which can be gridded in as  $0.4$  or  $.4$ .
60. *92. Probability & Statistics – Averages.* If Janelle's final average from 6 tests is to be 90, then the sum of her scores must be 540. If she already has 4 test scores that average 87, then the sum of those four tests is 348, so she still needs 192 points. If she scores the maximum 100 points on one test, then she can afford to score as low as 92 on the other test.
61. *48. Geometry & Measurements – Area & Perimeter.* If we let  $x$  = the length, then the width is  $3x$ . The area can then be represented as  $3x^2 = 108$ , which simplifies to  $x^2 = 36$ , so  $x = 6$ . This means the length is 6 and the width is 18, so the perimeter is  $2(6) + 2(18) = 12 + 36 = 48$ .
62. *54. Geometry & Measurements – Triangles.* This is a variation on the classic 3-4-5 right triangle where each side of the triangle is multiplied by 3. Since  $3 \times 3 = 9$ , and  $5 \times 3 = 15$ , we know that  $4 \times 3 = 12$ . Once we have this, simply apply the formula for area of a triangle:  $\frac{1}{2}bh$ , or  $0.5(12)(9) = 54$ .
63. *A. Algebra – Inequalities.* To isolate the variable, we subtract 1 from all three sides, giving us  $-2 \leq -2f \leq 8$ , and then divide all three sides by  $-2$ , resulting in  $1 \geq f \geq -4$ . That means that  $f$  can be any value less than or equal to 1 and greater than or equal to  $-4$ . On a graph, this is closed circles over  $-4$  and 1 and the number line shaded in between them.
64. *H. Algebra – Algebra in Context.* There are two different expressions provided, each of which describe how much a phone plan will cost, depending on the number of minutes spent talking,  $x$ . Set the two expressions equal to each other:  $50 + 3x = 60 + 2x$ . Combine like terms, and isolate the variable to one side of the equation:  $x = 10$ . Note that  $x$  represents the number of minutes spent talking. We can plug this back into either expression to determine the cost of a 10-minute conversation:  $50 + 3(10) = 80$ ; similarly,  $60 + 2(10) = 80$ . The companies charge the same amount of \$80.
65. *C. Numbers & Operations – Counting Principle.* Since there are four possible positions, and four different runners, there are  $4 \times 3 \times 2 \times 1 = 24$  different ways for the four runners to finish. We can abbreviate the runners with the letter of their first names and see that this is true. If William finishes first, then there are 6 possible ways for the other runners to finish in second, third, and fourth place. WXYZ, WXZY, WYXZ, WYZX, WZXY, WZYX. We could repeat the same thing to see what would happen if Xing Mei, Yuki, and Zack each finish in first place, but we can intuit that the results would be similar. No matter who finishes first, there are 6 ways for the other three runners to finish the race.
66. *F. Probability & Statistics – Averages.* Let the variable  $x$  represent Olu's score on the final exam. Her mean can then be represented as  $\frac{94 + 100 + 88 + 80 + x + x}{6}$ . We are adding up all the scores, with the final exam counted twice, and dividing by the number of scores. This expression can be simplified as  $\frac{362 + 2x}{6}$ . Olu wants her mean to be no less than 92, so we can make an inequality  $\frac{362 + 2x}{6} \geq 92$ . To solve this inequality, the first step is multiplying both sides by 6. We get  $(362 + 2x) \geq 552$ . Next, subtract 362 from both sides to get  $2x \geq 190$ . Finally, divide both sides by 2 to get  $x \geq 95$ . The lowest score she can get is 95.

67. B. *Numbers & Operations – Ratios & Proportions*.  $\frac{0.25}{24} = \frac{x}{60}$  so  $24x = 15$  and  $x = \frac{5}{8}$ .
68. E. *Geometry & Measurements – Circles*. Since Q is the center of the circle on the left, then PR represents the diameter, which we know to be 8. The radius, represented by PQ or QR, is half of this, or 4. The area of the circle with center Q =  $\pi(4)^2 = 16\pi$ . If this is 4 times larger than the area of the circle with center S, then the latter must be  $16\pi \div 4 = 4\pi$ . This means that the radius of the circle with center S can be determined by solving for  $r$  in the equation of the area of a circle:  $4\pi = \pi r^2$ . Therefore,  $4 = r^2$ , and  $r = 2$ , which is represented as RS or ST. Since the question asks for QS, and the circles are tangent (meeting exactly and only at point R), and we know that QR = 4 and RS = 2, then QR + RS = QS, and  $4 + 2 = 6$ .
69. D. *Numbers & Operations – Word Problems*. We can represent the information provided algebraically. Let  $x$  equal the number of times Ezekiel ran around the track. In terms of  $x$ , Darius ran  $\frac{1}{6}x$  around the track. We know that this is equal to  $2\frac{2}{3}$ , because the question tells us that Darius actually ran around the track this many times. So, we have an equation and solve for  $x$ :  $\frac{1}{6}x = 2\frac{2}{3}$ . We can rewrite both sides of the equation as follows:  $\frac{x}{6} = \frac{8}{3}$ . Simply cross multiply and solve for  $x$ :  $(8)(6) = 3x$  becomes  $48 = 3x$ , and  $x = 16$ .
70. F. *Geometry & Measurements – Angles*. Since the question asks for the average number of degrees in each interior angle of a decagon, we can use the formula for finding the degree measure of a regular polygon. Plugging in 10 for  $n$  in  $\frac{180(n-2)}{n}$ , we arrive at  $180(8) \div 10 = 144$ .
71. C. *Algebra – Algebraic Expressions & Equations*. To find the correct answer, try each answer choice by choosing any  $x$ -value and plugging it into each answer choice to see which results in the correct corresponding  $y$ -value. If we start by plugging in 0 for  $x$ , we find that  $y = x + 1$  and  $y = 2x + 1$  both work, but we can eliminate the other choices. If we move on and plug in 2 for  $x$ ,  $y = x + 1$  does not work, leaving us with  $y = 2x + 1$ .
72. H. *Numbers & Operations – Absolute Value*. If  $|9 - y + 6| = 21$ , then  $9 - y + 6 = \pm 21$ . Solving both equations gives us  $y = -6$  or  $y = 36$ . Since  $y < 0$ , then  $y = -6$ . Plug  $y$  into  $|x - 3 \times 5| = 6 + y$  and simplify to  $|x - 15| = 0$ . Thus,  $x = 15$ .
73. B. *Probability & Statistics – Probability*. Since it is not possible to pick out a part of a piece of chocolate, the probability of picking a piece of dark chocolate out of a bowl of candy must be a multiple of 6. If there were 11 pieces of candy in the bowl, and the probability of picking a piece of dark chocolate out of a bowl is  $\frac{5}{6}$ , then there would need to be  $5 \times 11 \div 6 = 9.16$  pieces of dark chocolate in the bowl of candy.
74. F. *Geometry & Measurements – Volume*. Use the formula for a pyramid to determine the volume of the glued-together figure.  $\frac{1}{3} 25(3) + \frac{1}{3} 25(6) = 25 + 50$ . In total, this becomes 75 cu in.
75. D. *Algebra – Plugins*. If  $b$  is doubled, then  $b^3$  is doubled 3 times, resulting in  $2 \times 8b^3$ . If  $c$  is doubled, we get  $\frac{2 \times 8b^3}{2c}$ , which simplifies to  $\frac{8b^3}{c}$  and represents an increase of four times over the original  $\frac{2b^3}{c}$ .
76. E. *Numbers & Operations – Fractions*. The total number of students polled is  $24 + 15 + 1 = 40$ . This means that  $24 \div 40$  students chose pizza. We can reduce this fraction by finding the greatest common



factor for both the numerator and the denominator. This would be 8, since  $8 \times 3 = 24$ , and  $8 \times 5 =$

40. Dividing both parts of the fraction by 8, we get  $\frac{3}{5}$ .

77. A. *Numbers & Operations – Factors, Multiples, Exponents, & Radicals*. Observe the proper order of operations. First, simplify the expression inside the parenthesis, which gives us  $4 + 3 = 7$ . Then, simplify  $\sqrt{16+9}$  to be  $\sqrt{25} = 5$ . Then, take the difference:  $5 - 7 = -2$ .
78. E. *Geometry & Measurements – Triangles*. Since points S and T are midpoints, triangle RST is also equilateral, and line segment PT creates two right angles – angle PRT and angle PTQ. This means that angles STP and TPS must each be  $30^\circ$ . Therefore, angle PST must be  $180 - (30 + 30) = 120^\circ$ .
79. A. *Numbers & Operations – Numbers*. If Bradley increased his speed by 2 miles per hour every minute for 15 minutes, then he increased his speed by a total of 30 miles per hour. If his starting speed was 40 miles per hour, then his speed at 12:15 p.m. was 70 miles per hour. If he was going 15 mph over the speed limit, then the speed limit was 55 mph.
80. G. *Numbers & Operations – Operations*. First calculate the value of the expression as it is, observing proper order of operations (PEMDAS). We first resolve the terms inside parenthesis so the formula becomes  $5 \times 2^2$ . We then resolve exponents:  $5 \times 4$ . Finally, we multiply to arrive at 20. Next, calculate the value of expressions without parenthesis. This expression would read:  $2 + 3 \times 4 - 2^2$ . We resolve exponents first:  $2 + 3 \times 4 - 4$ . Then we multiply:  $2 + 12 - 4$ . Then we add and subtract to arrive at 10. The difference between 20 and 10 is 10.
81. C. *Numbers & Operations – Percents*. Since Pete's total living expenses are \$1,200, and he spends 30% of it on rent, then his total rent expenses are  $\$1,200 \times 0.3 = \$360$ . Since he has 2 other roommates, there are 3 people in total who share the rent equally. Therefore, Pete's rent expense represents 1 of 3 or  $\frac{1}{3}$  of the total rent. Thus, the total rent is  $3 \times \$360 = \$1,080$ .
82. H. *Geometry & Measurements – Area & Perimeter*. The area of the piece of paper can be determined by the formula  $A = 2\pi rh$ , since this is the area of the cylinder without including the area of the top or bottom bases. We know that  $h = 8$  in. and  $r = 1.5$  in. Substituting these values into the formula, we find that  $A = 2(\pi)(8)(1.5)$ . This equals  $24\pi$ .
83. B. *Numbers & Operations – Percents*. Let  $x$  represent the original number. We can represent the fact that  $x$  was increased by 30% in two different ways:  $x + 0.3(x)$ , or simply  $1.3x$ . If from this point, the new number is increased further by another 30%. We repeat the calculations just performed, replacing  $x$  with  $1.3x$  instead:  $1.3x + 0.3(1.3x)$ , or simply  $(1.3)(1.3)(x)$ . Both simplify to  $1.69x$ , which means that compared to the original number  $x$ , the final number is 69% more.
84. G. *Geometry & Measurements – Coordinates*. We can use the midpoint formula to find the coordinates of point C. The formula is  $M = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$ . Substituting the coordinates of A and B into the equation:  $M = \left( \frac{2 + (-1)}{2}, \frac{2 + (-2)}{2} \right)$  and simplifying, we end up with  $M = \left( \frac{1}{2}, \frac{0}{2} \right)$ , or  $(0.5, 0)$ .
85. A. *Geometry & Measurements – Area & Perimeter*. To find the length of a side of a square, take the square root of the area. Since the area of  $WXYZ = 9$ ,  $\sqrt{9} = 3$ , and each of the four sides of the square has length 3. Since the area of square  $ABCD$  is 4 times larger than the area of square  $WXYZ$ , it must have a total area of  $4 \times 9 = 36$ . Thus, the length of a side in square  $ABCD = \sqrt{36} = 6$ . The difference between the length of sides in each square is  $6 - 3 = 3$ .
86. E. *Numbers & Operations – Imaginary Operations*. Substitute the values provided into the operation. Then, determine the value of each notation separately. We can express the first notation as  $((2 \times 6) + 4) \div 8$ . This simplifies to



$12 + 4 = 16$ , which we divide by 8 to get 2. The second notation can be expressed as  $((4 \times 7) + 2) \div 6$ . This simplifies to  $28 + 2 = 30$ , which we divide by 6 to get 5. The sum of 2 and 5 is 7.

87. *D. Numbers & Operations – Numbers.* If  $0 < y < 1$ , then  $y$  must be a positive fraction less than 1 whole. Going through each answer choice, we see that  $y^2 > y^3$  is true because every time you multiply a positive fraction by itself the value gets smaller.  $Y > 0.5y$  is also true because any positive value is always greater than half of its own value. Lastly,  $y > y^3$  is true for the same reason  $y^2 > y^3$  is true, meaning that all three inequalities are all true. We can prove this by substituting an example (i.e., a simple fraction like  $\frac{1}{2}$ ).
88. *H. Probability & Statistics – Averages.* A mean is simply the sum of all numbers in a range (list), divided by the total count of the numbers in that range (list). The question already describes the “sum of a list of integers” and the “mean of the integers.” The only other piece of information missing is the “count of numbers in that range.” We can use a simple example to prove this: if a range consists of the numbers 1, 2, and 3, then the sum of that range is 6, and the average of that range is  $6 \div 3 = 2$ . If we divide the sum (6) by the average (2), we arrive at 3, the count of numbers in the range.
89. *A. Numbers & Operations – Scientific Notation.* 10 raised to a negative power, like  $10^{-5}$ , lets us know that we need to move the decimal point “over” to the left a certain number of times. So, we move the decimal point to the left 5 times.
90. *F. Geometry & Measurements – Measurements.* Start by writing down the information provided, labeling the number line if needed. We must first determine the location of M and N. Since M is the midpoint of  $\overline{XY}$ , and since X and Y are located at  $-10$  and  $-2$ , respectively, we know that the midpoint of  $\overline{XY}$  is  $(-10 + (-2)) \div 2 = -6$ , so M is located at  $-6$ . Since N is the midpoint of  $\overline{YZ}$ , and since Y and Z are located at  $-2$  and  $8$ , respectively, we know that the midpoint of  $\overline{YZ}$  is  $(8 + (-2)) \div 2 = 3$ , so N is located at  $3$ . Therefore, the midpoint between M and N is  $((-6) + 3) \div 2 = -1.5$ .
91. *C. Geometry & Measurements – Measurements.* Since we start with Wesley’s shift at 6:00 a.m., we know that he must drive two more times after that to complete his third shift. If W stands for Wesley’s shift, and F stands for Wesley’s friends’ shifts, then the order of shifts will be WFFWFFW. If each shift is 7 hours long, and there are a total of 7 shifts, then Wesley’s third shift will be over 49 hours later. Since 48 hours is 2 full days, 49 hours is 1 hour more than 2 full days. Wesley’s third shift is over at 7:00 a.m.
92. *H. Numbers & Operations – Ratios & Proportions.* Since  $x$  is already being used, we’ll assign side JK another variable,  $y$ .  $\frac{3/4x}{12} = \frac{x}{y}$ . Cross multiply to get  $\frac{3}{4}xy = 12x$ . Divide both sides by  $\frac{3}{4}x$  to get  $y = 16$ .
93. *C. Numbers & Operations – Numbers.* If we set each of the answer choices equal to  $x^2 + 1$ , we can see which results in a nonzero integer value of  $x$ . For example, if  $x^2 + 1 = 16$ , then  $x^2 = 15$  in which case  $x$  is not an integer because 15 is not a square number. Going through each answer choice we find that only 82 works because if  $x^2 + 1 = 82$ , then  $x^2 = 81$  in which case  $x = 9$ .
94. *F. Geometry & Measurements – Measurements.* Using the formula for a midpoint, we take the sum of the two points and divide by two:  $(-17 + (-3.5)) \div 2 = -10.25$ .
95. *D. Numbers & Operations – Ratios & Proportions.* The ratio of the real tree to the painting is  $1 : \frac{2}{9}$  for the trees. Therefore, our proportion is  $\frac{1}{2/9} = \frac{x}{16}$ . Solving gives us  $16 = \frac{2}{9}x$ , and  $x = 72$ .



96. H. *Geometry & Measurements – Triangles*. The figure is divided into two right triangles: one on the left of the dotted line, and one on the right. On the right, we know the value of two of the three angles:  $30^\circ$  and  $90^\circ$ . Therefore,  $3y = 180 - 90 - 30$ , and  $y = 20^\circ$ . We can use this to solve for the measurement of  $x$  in the triangle on the left:  $x = 180 - 90 - 20$ . Therefore,  $x = 70^\circ$ .
97. B. *Numbers & Operations – Operations*. Resolve each exponential term so that the equation becomes  $\frac{-27 + 32}{16 - 1}$ . This simplifies to  $\frac{5}{15}$ , or  $\frac{1}{3}$ .
98. H. *Numbers & Operations – Percents*. The sales tax was \$6 and that was 8% of the original cost. Let  $x$  represent the original cost of the textbook. We can set this up using the proportion  $\frac{6}{x} = \frac{8}{100}$ . If we cross multiply this, we get  $8x = 6 \times 100$ . Dividing both sides by 8, we get  $x = 75$ .
99. D. *Algebra – Algebraic Expressions & Equations*. Solve for  $x$  by isolating it to one side of the equation, remembering order of operations, and the fact that what is done to one side of an equation must be done to the other side. First, divide both sides of the equation by 4 to remove the 4 in front of the factor  $x - 5$ . This gives us  $x - 5 = 7$ . Then add 5 to both sides, giving us  $x = 12$ .
100. G. *Numbers & Operations – Fractions*. After the first day of reading, the number of pages remaining is  $1 - \frac{1}{4} = \frac{3}{4}$ . The whole number 1 represents the total number of pages, whatever that is. We don't actually need to know how many pages there are, only that 1 represents the entire number of pages of the book, since we don't need to know how many pages remain to be read. After the second day, Latonya reads  $\frac{1}{3}$  of the  $\frac{3}{4}$  pages, meaning  $\frac{2}{3}$  of the  $\frac{3}{4}$  pages remain to be read. Multiply across the numerator and denominator for  $\frac{6}{12}$  and simplify to  $\frac{1}{2}$ .
101. B. *Geometry & Measurements – Angles*. Because lines W and X are parallel, and lines Y and Z are parallel, all angles formed by the intersection of the vertical and horizontal lines are corresponding. This means that the angles formed at each of the four intersections have the same relative angle measurements. For example, the top-left angle at each intersection shares the same degree measurement as  $q$ . In addition, the opposite angle (the bottom-right angle at each intersection) also shares the same degree measurement as  $q$  because they are vertical angles. Similarly, the bottom-left and top-right angles at each intersection share the same degree measurement as  $r$ . Therefore,  $q$  and  $r$  are supplementary angles adding up to  $180^\circ$ .
102. E. *Numbers & Operations – Word Problems*. First, determine the amount of money taken out of the piggy bank.  $12 \times \$0.25 + 9 \times \$0.1 + 6 \times \$0.05 + 5 \times \$0.01 = \$4.25$ . So, from the original \$4.31, we subtract the amount taken out (\$4.25) to be left with \$0.06 in the piggy bank. Of the given choices, the only way to do this is with 1 nickel and 1 penny.
103. C. *Geometry & Measurements – Circles*. A tire is a circle that rotates along its circumference. The circumference of a circle is  $C = 2\pi r$ . In this case, we know that the radius is 1 foot. Thus,  $C = 2\pi(1)$ , or  $2\pi$ , which is approximately 6.3 ft. In 2 minutes, the motorcycle will travel  $6,300 \times 2 = 12,600$  ft.  $12,600 \text{ ft.} \div 6.3 \text{ ft.} = 2,000$ .
104. E. *Numbers & Operations – Scientific Notation*. Divide the coefficients:  $9 \div 3 = 3$ . Divide the exponents and bases:  $10^6 \div 10^4 = 10^2$ . The resulting expression is  $3.0 \times 10^2$ .
105. D. *Algebra – Inequalities*. We first want to get rid of parentheses by distributing, so we get  $7 - 3x + 5 < 9x - 6 + 3x$ . Then we can combine like terms, resulting in  $12 - 3x < 12x - 6$ . We only want the variable on one side of the inequality, so we will cancel out the smaller variable by adding  $3x$  to both

sides, giving us  $12 < 15x - 6$ . Now isolate the variable by adding 6 to both sides, getting us  $18 < 15x$ , and dividing both sides by 15, resulting in  $\frac{18}{15} < x$ , which simplifies to  $x > \frac{6}{5}$ .

106. F. *Geometry & Measurements – Coordinates*. The given coordinates of  $(2, -3)$  are in the fourth quadrant, where  $x$  values are positive and  $y$  values are negative. The other end of the line segment must be in the second quadrant, where all  $x$  values are negative and  $y$  values are positive, since the line must pass through the origin (the center of the circle). The only coordinates that fit this criteria are  $(-2, 3)$  and  $(-3, 2)$ . If we were to plot these points on a graph, we would see that a line connecting  $(2, -3)$  with  $(-3, 2)$  does not pass through the origin.
107. A. *Algebra – Plugins*. To find out whether a proportion is true or not, you can cross multiply. If we do that to all four answer choices, we see that they all equal  $xy = wz$  except for the first choice.
108. F. *Numbers & Operations – Ratios & Proportions*. We can solve algebraically:  $2x + 3x + 4x = 180$ . Therefore,  $9x = 180$  and  $x = 20$ . But 20 is not the final answer because we are looking for the degree measure of the largest angle, which is  $4x = 4(20) = 80$ .
109. A. *Probability & Statistics – Probability*. If there are 21 marbles originally, and 5 are removed, then 16 remain. The question states that after the 5 marbles are removed, the probability of picking out a brown marble is 50%. This means that the total number of brown marbles must be exactly half of the total number of marbles that remain. Since 16 remain, the number of brown marbles must be 8, which is the same as the original. Thus, no brown marbles were removed.
110. F. *Geometry & Measurements – Measurements*. If Maritza always reads at a rate of 1 page per minute, then in 15 minutes, she will have read 15 pages. Since the total number of pages in the magazine is 25, then  $25 - 15 = 10$  pages remain. As a fraction, this is  $\frac{10}{25}$ , which simplifies to  $\frac{2}{5}$ .
111. D. *Probability & Statistics – Averages*. If we put the nine given numerical values in order, we have 1, 5, 5, 5, 9, 9, 9, 9, 11. Including the tenth value, the only way for the median to be 7 is if the two middle values are 5 and 9. However,  $x$  cannot be 5 because then the mode would be 5 and 9 instead of just 9.
112. F. *Geometry & Measurements – Area & Perimeter*. The area of a parallelogram is equal to  $A = bh$ . So the area is equal to  $A = 12 \times 18 = 216$ .
113. A. *Geometry & Measurements – Angles*. The total degree measure of the interior angles of any polygon with  $n$  sides can be found using the formula  $180(n - 2)$ . Since we are given the total degree measure, we can set the formula equal to the known amount.  $1,800 = 180(n - 2)$ . Isolate and solve for  $n$  by dividing both sides by 180, then adding 2 to both sides. This gives us  $10 = n - 2$ , then  $12 = n$ .
114. G. *Algebra – Algebraic Expressions & Equations*. Like the above problem, we can find the smallest integer in the group by setting up an equation:  $x + x + 2 + x + 4 + x + 6 + x + 8 = 110$ , which simplifies to  $5x + 20 = 110$ , resulting in  $5x = 90$  and finally  $x = 18$ . However, that is not the final answer because we are looking not just for the smallest integer but the sum of the three smallest integers, which is therefore  $18 + 20 + 22 = 60$ .

## Stand-Alone Revising/Editing Skills

### Grammar Glossary

-  **Sentence** – a sentence must have a predicate (most generally, a verb) and a subject (of that predicate/verb). It may also have other words that help to form a complete thought.
-  **Subject** – usually a noun or pronoun; the person, place, or thing that the sentence is actually about

- Predicate – what the subject of the sentence is doing
- Clause – a group of words that has a predicate and subject
  - Independent clause – a clause that can also stand alone as a sentence (a clause that also contains a complete thought)
  - Dependent (subordinated) clause – a clause that cannot stand alone as a sentence (a clause that does not contain a complete thought)
- Phrase – a group of words that does not contain the sentence’s verb or subject
  - Prepositional phrase – a group of words beginning with a preposition but ends with a noun (acts like an adjective, or adverb)
  - Participial phrase – phrase beginning with a participle (a verb that ending in –ing, which acts like an adjective or adverb)
- Fragment – a group of words that lacks any one or more of the three elements that form a sentence; sometimes a dependent clause written as if it was an independent clause.
- Run-on – two independent clauses joined together without appropriate or sufficient punctuation and/or conjunctions
  - Fused sentence – a run-on sentence resulting from the lack of any punctuation
  - Spliced comma – a run-on sentence resulting from the inappropriate use of a comma to join two independent clauses
- Compound sentence – two independent clauses joined together with appropriate or sufficient punctuation and/or conjunctions
  - Conjunction – a word that joins two clauses together; frequently preceded by punctuation
    - Coordinating conjunction – connects two clauses of equal importance (i.e. two independent clauses); *and, but, or, so, for, nor, yet*
    - Subordinating conjunction – connects a dependent clause to an independent clause; *after, before, if, since, while, etc.*
    - Correlative conjunction – pairs of words that signal a relationship between two elements in a clause (i.e. either/or, both/and, neither/nor)
- Verb Tense – at its most basic, the different forms of a verb that indicate when an action takes place (i.e. in the past, present, or future; whether something continues to happen)
- Verb Mood – indicate the author’s attitude in terms of what the intention is: to show a statement of fact (indicative), issue a command (imperative), or propose a statement contrary to fact (subjunctive)
- Number (verb, noun) – indicates whether a verb or noun is singular or plural
- Preposition – a word that links a noun to another word, often describes the position of that noun (i.e. in, etc.)
- Voice – refers to the way a sentence is written where the subject performs the action indicated by the verb (active) or where the verb acts upon the subject (passive)

## Pronoun Clarity

1. A. As written, “her” could refer to either the manager or the employee; so too could “his” refer to both the employee and the manager, since we do not know either’s gender. Using “a” helps to clarify the ownership of the bonus. This question is an example of a pronoun with more than one possible



antecedent. We do not refer to people as “it.” Remember that “its” is possessive, and “it’s” is not (it’s a contraction for “it is”).

2. H. In this sentence, the word “they” could refer to the people who do the shopping, the actual after-dinner sales, or even the dinners themselves – another example of a pronoun with more than one possible antecedent. “These,” “those,” and “some” do not clarify the sentence any more than “they.” Only by repeating “the sales” in the dependent clause after the second comma in the sentence does the sentence become clear. Alternatively, “the former.”
3. D. The pronoun “he” is unclear: is the sentence referring to the stranger, or to the agoraphobic man? In order to clarify, the best choice is “the former,” as “anyone” and “it” are as vague or inappropriate. Pronouns must have a clear antecedent to which they refer. In this case, “former” refers to “stranger,” letting the reader know who spoke the sentence. “Him” results in the improper use of the pronoun.
4. G. The pronoun “she” could apply to either Sarah or Melissa, so the sentence must clarify who had made the bigger mess. In this case, the word “bigger” means that one person made a mess larger than the other, so “they” does not work to replace “she.” “Melissa’s” is possessive, not a contraction for “Melissa has”.
5. A. An antecedent need not always precede the pronoun. In this sentence, the pronoun “it” refers to the extra water, which needs to be pumped out of the ship. “He,” “they,” and “those” are not appropriate pronouns to refer to the water. In this case, “it” clearly refers to “extra water,” since we don’t refer to people as “it.”
6. G. It is not clear whether “they” refers to the teachers or the students. Based on the context, it can be inferred that it is meant to refer to the students. Therefore, the pronoun should be replaced with the noun itself (“the students”) for clarity.
7. D. There is no pronoun in sentence 1. The “he” and “his” used in sentence 2 clearly refer to Jordan, a proper noun (particular person). Similarly, in sentence 3, though we now have Jake, another proper noun (particular person), the “he” clearly refers to Jordan. It is not until sentence 4 that we have an unclear pronoun; we are unsure who “he” is – is the pronoun’s antecedent Jake or Jordan? Is Jake suggesting that Jordan go to the doctor, or is Jordan himself making that suggestion?
8. F. Clyde and Billy are mentioned in sentence 1, but sentence 2 begins with “he,” so we are unsure of who made the suggestion to jackhammer through the vault. It is clear from sentence 3 that “they” refers to Clyde and Billy.

## Modifiers

1. D. The word “quickly” is a squinting modifier (a modifier placed in such a way as to be able to modify multiple ideas in the sentence). As written, we can interpret the sentence in two different ways: either “reading at a greater speed” improves one’s vocabulary, or “reading a greater quantity” improves one’s vocabulary at a greater speed. Only by moving “quickly” to the end of the sentence does the sentence become clear: that reading a greater quantity (“reading more”) can improve one’s vocabulary more rapidly. The other revisions do not address the issue of ambiguity.
2. F. As written, the sentence begins with a dangling modifier – one that modifies nothing in the sentence as written. In this case, the modifier is “While playing the lead in *Hamlet*”. Without it, the sentence is grammatically correct, yet with it, the sentence is confusing and ambiguous: was the audience groaning as the actor played the lead in *Hamlet*, or was the audience – for some reason – playing the lead in *Hamlet* and groaning simultaneously? This choice correctly places the modifier close to the modified thing (the actor) and clarifies the sentence.
3. C. The dangling modifier “shining from a new coat of car polish” inappropriately modifies Jeremy, but should modify car. Therefore, we should move it after “his car” and offset the modifier with two

commas, one before it, and one after it. The other choices do not clarify the ambiguity, as moving the modifier to the end of the sentence or directly after “Jeremy” results in an equally nonsensical sentence.

4. H. The modifier “dripping with anticipation” could be deleted and still result in a grammatically correct sentence. However, deleting or switching the other modifiers in the sentence would still leave the sentence ambiguous: is it Corey or the kitten that is “dripping with anticipation”?
5. D. There are many modifiers in the paragraph (“on the last day of February”; “which he had received for his birthday”; “nearby”). However, it is not until “collecting into tiny pools” that we find a modifier out of place. As written, it sounds like Forrest himself is “collecting into tiny pools” when from context we find that the modifier should actually apply to the melting icicles. One way to correct this would be to write “Forrest watched the icicles hanging from the trees slowly melt onto the sidewalk, collecting into tiny pools.”
6. F. There are modifiers throughout the paragraph, but only in the second sentence is the modifier misplaced. In sentence 2, “with a grudge” describes how Amber signed the agreement. Instead, as written, it suggests that Amber signed an agreement literally with the other party being “a grudge.” One way of clarifying this would be to write “As a result, Amber grudgingly signed the unfavorable agreement.”
7. A. To find a modifier, remove a word or phrase that describes how, when, why, or what happened to something else in the sentence, and see if the sentence can stand alone as a complete thought. In this case, the phrase “after the science lesson” describes when something happened. However, we don’t know what this applies to: is it when Ms. Cortez said something, or when the students would be able to enjoy a break? Depending on the intended meaning, the sentence could read “Ms. Cortez said that the students would be able to enjoy a break after the science lesson.” Or, it could read “After the science lesson, Ms. Cortez said that the students would be able to enjoy a break.”
8. F. In sentence 2, the placement of the adjectives “crowded and uncomfortable” is such that they appear closer to the nouns “house” and “Florida,” and could therefore be misconstrued to be describing those nouns. Since sentence 3 goes on to tell us about how uncomfortable the bus is, the modifiers should actually refer to the bus and be moved closer to “bus”.

## Tenses, Mood, & Number

1. A. Verbs must agree in tense, and tenses relate to time. The three main verb tenses relate to the past (something that happened in the past), present (something that is happening now), and future (something that will happen in the future). Each of these three tenses are further divided into the simple, progressive, perfect, and perfect progressive tenses, which each indicate when something has been (or is being or will be) done. In this sentence, the verbs “realized” and “searched” are written in the past simple tense. Therefore, the verb “jump” should also be written in the past simple tense as “jumped.” The other forms of the verb “jump” are written in various forms of the present or future tense.
2. H. The sentence suggests that the counselors had already found Tanya, which means that Tanya must have already hidden after hearing the scary story and subsequently been found by the counselors. Since this is the case, neither the future simple tense “will hide” nor the future perfect tense “will have hidden” make sense; neither does the present perfect progressive form “has been hiding.” Notice that “is hidden” works like an adjective, not a verb. Only the past perfect progressive form “had been hiding” makes sense, as the hiding has already taken place and describes how Tanya had hidden for some extended period of time.
3. C. As written, the tense of the first half of the sentence does not match the second half of the sentence. The sentence is referring to an event in the future, so the verb tense should be changed to future tense: Alicia “will have” to study hard all week.

4. E. In addition to tenses, verbs can also be classified by mood. There are three moods: the indicative, the imperative, and the subjunctive. The vast majority of verbs are used in the indicative mood, which states a fact or indicates a state of being. The imperative mood is used to indicate a request or a command. The subjunctive mood is used to express a wish or state something contrary to fact. In this sentence, the verb “call” is written in the imperative mood, telling someone (the “your” in the sentence) to make a phone call. “Tell” expresses the verb “tell” in the imperative mood. The other choices are written in the indicative mood, including “you tell.”
5. C. The subjunctive mood (only used with the verbs “be” and “were [to be]”) are used to show something contrary to a fact or to express a wish or request. Since the author is not likely to be a fictional character in reality, the person the author addresses is not likely a billionaire. The indicative verbs “was,” “is,” and “are” are not appropriate to use. The phrase “would be” is often a clue that the verb in question should be in the subjunctive mood.
6. H. It may appear that no sentence is affected by a verb in the wrong mood. Upon closer inspection, sentence 4 describes a state of being that currently is not the case (i.e. that all teachers and students should attend the assembly). Notice that what is described calls for the subjunctive mood, as it describes a situation “as it should be.” This means the verb “are” should be changed to “be.” “Are,” “is,” and “being” are all indicative moods.
7. A. When an action is being performed by the subject on objects, then that is called active voice. When the sentence is written so that the objects are being performed on by the subject, then that is called passive voice. Along with choices C and D, the subordinated clause (“as...him”) is written in the passive voice where the independent clause (“Brent...routine”) is written in the active voice. To correct this, this choice uses the present simple tense of “treat” (which is congruent with the present simple tense of “make”) to show that the action is being performed on the beans, water, and implements by Brent.
8. G. In every sentence but number 3, the verbs all use the appropriate tenses to describe the ideas in the sentence. In sentence 3, we are told that Dominic “will have been continuing” to invest based on the fact that he “had been receiving” high returns. The sentence tells us that what happened in the past will affect the future. However, “will have been continuing” is written in the future perfect progressive form, where it requires the future progressive form.

## Possessive Nouns & Determiners

1. C. In the case of nouns referring to more than one person, place, or thing (nouns with a number greater than one), it is important to add an apostrophe after the pluralized ending of the noun or proper noun. In this case, “waitress” refers to a single person, while “waitresses” refers to a group of people. Generally, an “-’s” is added to a noun to indicate possession. However, in the case that the noun ends in an “s”, the extra “s” is omitted. The possessive form of “waitress” is “waitress’” (note the apostrophe, but no additional ‘s’), and the possessive form of “waitresses” is “waitresses’” (again, the same rule for the apostrophe). We know that we are talking about more than one person because the sentence says “they” earlier on.
2. G. Though we generally add an “-’s” at the end of a noun or proper noun to indicate possession, some pronouns are understood to indicate possession. The word “his” is the possessive form for the pronoun “him” and does not require an apostrophe to indicate possession. There is no such word as “his’” or “his’s”. In this case, “its” is inappropriate because we are referring to a person, not a thing. “Their” is the possessive form for “they,” or more than one person, though we are only talking about Dexter in this case.
3. C. Within the context of this sentence, it can be determined that the slipper (or “it”) belongs to Cinderella. Therefore, Cinderella’s name should be made possessive. In order to do this, an apostrophe and then an “s” should be added to the end of her name to indicate singular possession.

4. E. The possessive form of the pronoun “she” is “hers” and similarly does not require an apostrophe to indicate possession. There is no such word as “her’s”. “She’s” is a contraction for “she is.” Note that “her” is also possessive, but is a possessive adjective, not a pronoun.
5. A. The word “its” is the possessive form of the pronoun “it.” Remember that “it’s” is a contraction (an abbreviated form) of “it is”. The apostrophe DOES NOT indicate possession.
6. H. As with pronouns that represent a single noun or proper noun, pronouns that represent more than one noun or proper noun do not require an “-’s”. The sentence compares one person’s responsibility with some unknown number of other people’s responsibility. The pronoun for “other people” would be “them/they.” The possessive form of this is “their/theirs” (depending on whether or not the object is left out of the sentence).
7. A. Sentence 2 uses the possessive pronoun “hers” to tell us that it is Camilla’s closet that is filled to overflowing. This is the possessive form for “she,” which as a pronoun, refers to Camilla, in this case. “She” is a different pronoun which refers to Camilla. Notice that sentence 3 uses the possessive adjective “her” to tell us whose favorite item was the beret. Sentence 1 simply omits a “-’s” to indicate possession. Instead, the sentence should be begin with “Camilla’s preference...”. This tells us that it is Camilla who has the preference
8. G. We can see several examples of the possessive form of plural nouns and pronouns being used. In sentence 1, “people” is the noun representing more than one person, and “people’s” is the possessive of this noun. “Those” in sentence 2 tells us that it is the homes’ value that have recovered. In sentence 4, “their” refers to Cassidy and Kaleb. The subject of sentence 3 requires an apostrophe to show that it is their home that lost 50% of its value. The subject, in this case, is two proper nouns: Cassidy and Kaleb. We’re told that the two of them purchased the house together. Therefore, they have joint possession, and the “-’s” should come after the last of the nouns. The sentence should begin with “Cassidy and Kaleb’s home...”. As written, the sentence tells us that the home is “Cassidy’s,” not Kaleb’s.

## Subject-Verb Agreement

1. A. Singular subjects must be matched with singular verbs, as plural subjects must be used with plural verbs. The matching of singular subjects with singular verbs and plural subjects with plural verbs is the act of ensuring subject-verb agreement, or ensuring that the subject and verb numbers agree. Identifying the subject is critical. The boys, the lake, the time, and the town are all nouns, but only the camp is the subject. The prepositional phrase “for girls” describes “camp” while the entire phrase “some five hours away from the closest town” describes the general location. The subject, therefore, is “camp.” The verb “are” is plural, and the subject “camp” is singular. Therefore, the only correct choice is to use the verb “is.”
2. F. The verb “was” must be congruent and match in number with the subject (“Doris”), which is singular in number (there is only one Doris). Therefore, the only singular verb of “be” is the past tense verb “was”; “have been” and “were” are both plural (i.e. “they were” or “they have”). “Is” and “will be” are also singular, but in the wrong tense.
3. A. The subject of this sentence, “cuisine,” is a singular noun. The verb, which as the sentence is written is “are,” should therefore be singular. The object, which is “Texas and Tennessee,” may mislead readers because it is plural. In order to match the subject, the verb should be changed from “are” to “is” to be singular and present tense.
4. F. Whenever the word “every” appears in front of a word or group of words, the verb associated with those words should be singular. “Was” is the only singular answer choice. Changing “participate” to “participates” does not fix the problem of agreeing the verb “to be” with the subject “every member.”



5. A. Certain indefinite pronouns such as “everybody” may seem like they should be plural (since it does talk about more than one person), but they are actually singular. Therefore, only the singular verb “was” is appropriate here (think of “everybody” as a single unit).
6. H. Compound subjects (in this case, “financial functions and operational functions”) that are joined by the word “and” take a plural verb (in this case, “are”).
7. C. Some compound subjects that are joined by the word “and” are actually treated as singular if they are thought of as a single unit. In this case, the friend is also the coworker, and should be accompanied by a singular verb such as “schedules.” We know this because the following sentence tells us that there are only two people involved.
8. E. When a plural subject and a singular subject (“Grace’s three daughters” and “her son,” respectively) are joined by “or” or “nor,” the number of the verb follows the number of the subject closest to it. In this case, “Grace’s son” is closest to the verb of being, “is/are/was/were,” etc. Thus, we should choose the singular verb of being: “was.” We could write the sentences out separately to prove this. 1) “Even though it was bedtime, Grace’s three daughters **were** not ready for bed.” 2) “Even though it was bedtime, her son **was** not ready for bed.” So, “are” should change to “is.”

### Frequently Confused Words

1. C. To “meddle” is to interfere with something. “Medal” (an award or commendation) and “metal” (a hard, material made of certain minerals) are tangible objects; only “mettle” refers to the intangible resolve of the team. All three of these words are homophones. Note that “muddle” means to confuse something.
2. H. There are three homophones in this question: “whether,” “weather,” and “wether.” Only the former expresses doubt about the students’ ability to pass the test. “Wether” is not a real word, and “weather” refers to the atmosphere in a particular place. “Solely” means “only,” and is far from the meaning of “soulfully,” which means to do something with passion or emotion.
3. D. “Bare” and “bear” are homophones. To “bare” something means to uncover it, whereas to “bear” something means to carry, support, or endure something (unless, of course, we’re talking about the large, furry mammal; context dictates the meaning). Note that another homophone is used in the sentence: “accept” and “except”. To “accept” something means to consent or believe it, where “except” specifies that something should not be included. In this case, Jamie could not believe, she was going to “actually” jump out of a plane.
4. G. “Off” is an adverb/preposition that describes how something is removed or separate from something else. “Of” is a preposition that describes the relationship between some part of a whole. In this case, “off” is used to describe how one might move away from a route. The homophones “course” and “coarse” have different meanings. The first describes a path or route, while the second describes somethings that is rough. Similarly, “threw” is the past tense of “throw,” where “through” describes the action or motion of going from one side of something to another.
5. A. The word “new” refers to something that is novel or not seen or experienced before. The word “knew” is the past tense used to describe how something was understood. There are other homophones in the other sentences (“real” and “reel”; “flower” and “flour”), but they are all used correctly in context.
6. F. Based on the context, it can be determined that the word “idle” is meant to be a noun and describe someone Gerard is interested in meeting. As it is currently spelled, “idle” is an adjective that means someone is not moving. Therefore, it should be spelled “idol,” which means a person or figure that is greatly admired.
7. B. There are many homophones used through the paragraph (“week” and “weak”; “horse” and “hoarse”; “suite” and “sweet”; “vein” and “vain”). However, all of the context-appropriate words are

used except for “horse,” which is a noun referring to the four-legged animal. Instead, the word should be “hoarse,” which is used to describe a person’s voice which sounds rough or scratchy.

8. H. Sentence 2 requires a revision. “There” and “their” are used correctly, but “wave” (the motion) is not. Instead, “wave” should be written as “waive,” which is a verb that describes the refraining from enforcing a rule. In sentence 4, “their” should be “there.”

## Logical Comparison

1. D. The noun being compared is “weight.” The other answer choices all compare the dire wolf’s weight to domesticated lapdogs themselves, not the weight of the domesticated lapdogs. Only by using “that of a” is it clear that we are comparing weights, and not an attribute of an object with an object itself.
2. H. The noun being compared is “taste” or “quality” (of pizzas), though this is not made explicitly clear in the sentence. Simply adding an “-s” to “Chicago” does not help clarify that this is the case, since the sentence could be read to compare NYC’s pizzas with NYC’s pizzas “that belong to Chicago.” This is clearly nonsensical. Instead, we are talking about pizzas made in NYC as compared with pizzas (“those”) made in Chicago.
3. B. The noun being compared is “ability”(or skill, to make delicious sauces). The sentence as written compares one person’s ability with “anyone else,” not the skill or ability of “anyone else.” Only by saying “anyone else’s skill” is this clear. The other choices make a similarly erroneous comparison, comparing skill (the attribute of a person) with a person itself.
4. G. The noun being compared is the day’s newspaper (yesterday’s versus today’s). The only choice that makes this clear explicitly says “yesterday’s cover.” The first two choices change the meaning of the sentence, but also compare the cover of today’s paper with the other day itself.
5. C. The noun being compared is “style.” Other nouns in the sentence might be misconstrued as being compared, including “Matisse/Picasso” or “paintings.” As the sentence is currently written, it compares Matisse’s style directly to Picasso. The sentence should be changed so that the two painters’ styles are being compared, and therefore the option of changing “over Picasso” to “over the style of Picasso’s art” is the best option.
6. H. The comparison made in sentence 4 is illogical, since it compares the noun “amount” with “everyday glassware.” The true comparison being made is between the amount in one thing versus the amount in another, not the amount in one thing versus another thing itself. Note that this is the only sentence in which a comparison is being made.
7. C. In this paragraph, sentences 2 and 3 both contain logical comparisons. In sentence 2, the quality of “bagels” at “one shop” are compared to the quality of “bagels” at “another [shop]”. This is a logical comparison. In sentence 3, however, a comparison is made between “the West Side” (a place) and “living on the East Side” (an action). This is illogical, as the comparison should be between “the West Side” or “the East Side” (or alternatively, “living on the East Side” versus “living on the West Side”).
8. F. The paragraph is concerned with comparing the amount of potassium in a variety of different foods. Sentence 2 compares “the amount of potassium” in kiwis with “bananas.” This doesn’t make sense. Instead, it should compare “the amount of potassium” in one food with “the amount of potassium” in another, which is the case in sentences 3 and 4.

## Subordination & Coordination

1. C. Conjunctions are used to join two related ideas together to form a complete, logical thought. There are two different types of conjunctions: coordinating conjunctions and subordinating conjunctions. The first refers to conjunctions that typically connect two independent clauses/sentences that are related in idea. The second refers to conjunctions that connect an independent clause with a dependent (subordinated) clause (an idea that cannot stand alone as a sentence). There are two separate but related ideas in this sentence. The first is a statement that products with more reviews sell more than those with fewer reviews. The second is that retailers desire that shoppers leave reviews. These ideas are related – they have to do with product reviews. So, we can connect the two with an appropriate coordinating conjunction: but, or, yet, so, for, and, nor. Of these choices, only “so” indicates a cause/effect relationship. In this case, it makes sense that stores would want to have shoppers leave reviews for products, since those products sell more than those without reviews.
2. H. There are two independent clauses (“The car...yesterday” and “It nevertheless...supermarket”) that discuss the same idea – a broken car needing repairs. The clauses are joined by the coordinating conjunction “for,” which is similar to “because” in that it shows a positive cause/effect relationship (one thing led to another thing). However, this is illogical, since the sentence as written states that it was taken to the mechanics to be repaired yesterday *because* (for) it broke down today. Since a car that had just been fixed should not break down again the next day, the appropriate conjunction to use is “yet,” which describes a situation contrary to expectations.
3. D. A sentence must express a complete thought. In this case, the sentence is made up of two parts: an independent clause and a dependent (subordinated) clause. Dependent clauses are “dependent” on something else – they tell about things elsewhere in the sentence; for example, when, how, or why something happens. In this sentence, “The street performer...tourists” is an independent clause because it is a complete thought by itself. The phrase “during juggling six bowling pins and a sword” is a subordinated clause because it tells more about something in the independent clause, and cannot stand alone as a sentence on its own. In this case, we can tell that the dependent clause describes when something happened. However, “during” is used to describe a noun (for example, during the juggling act). In this case, no noun is described (an action – juggling – is described), and the appropriate conjunction to use to show subordination is “while.” “Whether,” “whenever,” and “because” require another noun (for example, “Whenever he juggled six bowling pins and a sword...”).
4. E. A special type of conjunction is called a correlative conjunction, and is always used in pairs. These are “both/and,” “either/or,” “neither/nor,” “not only/also,” and “not only/but also.” They show a relationship between different clauses. In this case, the sentence describes a choice between two things, so we must use “either” and “or” not “either” and “and.”
5. B. As described in the previous answer explanation, there is particular correlative conjunction required for this sentence. The word “neither” is used to describe the relationship between “chicken” and “fish” in the sentence. Therefore, the conjunction should be “nor.”
6. F. There are many conjunctions used throughout the paragraph (“and,” “though,” “for,”). In sentence 3, “for” is used incorrectly, as it is similar to “because” in that it describes how one thing was the result of another. It doesn’t make sense to say that her parents were more concerned than upset as a result of Amie’s expecting them to be very angry. Instead, since the parents’ reaction was not as Amie expected, the right word to use would be “but” or “yet.” These conjunctions are coordinating conjunctions. In sentence 2, “though” is used incorrectly. It doesn’t make sense that her parents cared about her grades “though” they always asked... In this case, “since” would make more sense.



7. D. The subordinating conjunctions “whenever,” “as soon as,” “so that,” and “though” are used in the paragraph. However, “though” is used incorrectly because sentence 4 describes two similar situations where not having a clear storyline would negatively impact other departments. In this case, we’re told that the casting department might pick the wrong actors, and the costume department might pick the wrong styles of clothing. Since these two are related (and are joined by “so too,”) we know they must be logically equivalent. This means we must change “though,” which implies a contradiction of some kind, to be something else (like “just as”).
8. F. Every sentence uses a correlative conjunction. Sentence 1 uses “both/and,” sentence 3 uses “neither/nor,” and sentence 4 uses “either/or.” Sentence 2 tries to use “not only/but also,” but inappropriately uses “not only/and to also.”

## Within-Sentence Punctuation

1. B. Commas are used to separate a series of items, to separate adjectives, to help conjunctions join two independent clauses, to signal quoted material (along with quotation marks), and to otherwise set apart clauses and phrases. Colons do not function like commas, and are used to signal a list. A semicolon is used to join two logically connected independent clauses together. In this case, “friends and family” is not an independent clause (it is incapable of standing alone as a sentence), so a semicolon is inappropriate. The sentence indicates that there are two things needed for Jordan to be happy, and a colon is best used to signal that a list of these things follows.
2. F. Commas are used at the end of an introductory adverb clause, in this case “Because she had recently moved to a new town.” Adverb clauses are a group of words with their own subject and verb that together act like an adverb, describing a verb, adjective, or another adverb. A semicolon is inappropriate, as the adverb clause cannot stand alone as a sentence. Neither is a colon appropriate, since the phrase after it is not a list of items. There is no need to offset “recently” since it gives important information about when Melissa moved.
3. D. There are two independent but illogically related clauses in this sentence. Droughts in the American Southwest have little to do with umbrellas in India. Because these ideas are unrelated, they cannot be joined with a semicolon. Using a comma results in a comma splice. A colon is also inappropriate to use given the content of the two clauses. Thus, only by splitting the sentence in two can the sentences be properly punctuated.
4. E. Commas are used to separate two or more adjectives that describe the same noun. In this case, “sleek” and “stealthy” describe the cat and should be separated by a comma. This is not accomplished by adding a comma in any other suggested locations.
5. C. One of the ways a hyphen (not a dash) is used is to denote a compound adjective, which is a situation where two or more adjectives form a single image or thought that describes a noun. In this case, “sixteenth-century” is used as a compound adjective to describe the type of scientist that Galileo was. If we were simply referring to the “sixteenth century,” no hyphen would be needed, because “sixteenth century” is a noun (a period of time).
6. H. “If they could get a ride” is not an independent clause, so neither the semicolon nor a period are appropriate. A dash is used to introduce an interruption to the original thought, to emphasize a previously mentioned thought, or to expand further on something mentioned in the sentence. The main purpose of the sentence is to tell about how Seth wants to go out with Travis. A dash is used to provide additional information that complements the main purpose of the sentence – namely, a condition that must be met. A compound adjective, such as “three-day”, is used to describe a noun (“weekend”), and must be hyphenated. No commas are needed in sentence 2, as “he” (Seth) is the subject that does both actions: “stay” and “play” (this is a compound sentence where one subject



does two actions). A comma is always needed after a subordinate clause (“however”) as well as to separate extra information (“Travis”) from the rest of the sentence.

7. B. Commas are only necessary in certain circumstances. The comma in sentence 4 is necessary since it connects the conjunction “now” with the rest of the independent clause. The commas in sentence 3 are similarly necessary, since they offset clauses that tell more about why the investigators drew the conclusion that they did (the sentence is a compound sentence and includes a subordinated clause “Since...area”). Similarly, the comma in sentence 1 is used to offset the subordinated clause “after...extinguished” from the independent clause “the firefighters...happened.” In sentence 2, however, it is not used correctly, and should be removed. This is because “The investigators...corner” is not an independent clause – it can’t stand alone as a sentence. It needs the verb “caught” in order to work.
8. F. In sentence 2, a comma is used to introduce a list, which makes the sentence a run-on. In order to clearly indicate the start of the list, a colon should replace the comma.

### End-of-Sentence Punctuation

1. D. The sentence is an interrogative sentence (that asks a question) and should be punctuated with a question mark. Many interrogative sentences begin with similar phrases that use words such as: “who, what, where, when, why, how, do.” Exclamation points are used to punctuate exclamatory sentences (that express strong emotion). Periods are used to punctuate declarative sentences (that make a statement) or imperative sentences (that give commands). It is never appropriate to use two forms of punctuation to end a sentence.
2. F. The sentence is a declarative sentence and should be punctuated with a period. A question mark is used to punctuate an interrogative sentence. It is never appropriate to use two forms of punctuation to end a sentence.
3. C. The sentence is not a question and should not be punctuated with a question mark. The sentence could either be declarative or exclamatory, but only one option is presented (the one using an exclamation point). It is never appropriate to use two forms of punctuation to end a sentence.
4. F. While declarative and imperative sentences (punctuated with a period) are always included before the closing quotation marks, question marks and exclamation points can either be included before or after the closing quotation marks, depending on the content being quoted. If the quoted material asks a question or is itself exclamatory, the question mark and exclamation point go before the closing quotation marks. If the sentence itself (not the quoted material) is a question or an exclamation, the punctuation goes after the closing quotation marks. In this case, the quoted material is a question – the sentence itself is not a question. When concluding a sentence with a question mark or exclamation point, no other punctuation is necessary.
5. C. This sentence is actually a question, quoting a statement (not a question) made by Allison. Because of this, the question mark does not go inside the quotation marks, but after it. No other punctuation is necessary.
6. F. Sentence 2 is actually a question, asking “Who knew...”. Therefore, it requires a question mark, not an exclamation point.
7. C. Sentence 3 talks about questions, but is a statement of fact. Since it is a declarative statement, it should be punctuated with a period. Note that in sentence 4, it is appropriate and permitted to use question marks to punctuate fragments if there is a series of questions. No capitalizing of the fragments are necessary, since they are understood to be extensions of the same “base question” (which in this case is “What should the students pack...”).
8. G. The dialogue in sentence 3 should end with a question mark because it is a question: “Why didn’t we get in line sooner?” The speaker “wails” when speaking the line, so an exclamation mark might

seem appropriate, but it is grammatically incorrect. Therefore, the exclamation mark should be replaced with a question mark.

## Punctuating a Series

1. D. As it is written, the only acceptable choice uses a comma. A semicolon or period results in a fragmented second-half of the sentence. A colon is not used appropriately since the items in a series are already denoted as such with the use of the phrase “such as the.”
2. G. Semicolons are often used to separate items in a list (as signaled by the use of a colon in “places: Seattle”) that are already punctuated with commas. By leaving a comma between “Washington” and “Chicago,” it would be unclear if the author meant that he or she lived in four places as opposed to three. By using a semicolon after each location, it becomes clear that the author had lived in three places only. A city and state are properly separated by a comma (as in “Seattle, Washington”).
3. D. We cannot leave the semicolon or colon in the sentence. The semicolon creates two fragments. Leaving the colon and changing the semicolon into a comma creates a list of four things: three names, and the rest of the sentence (which doesn’t make sense to include in the list). We can’t change the colon and semicolon to commas, because what results is a confusing sentence (did Cheryl, Blake, and Daniel argue with three coworkers, or were Cheryl, Blake, and Daniel the three coworkers themselves?). Instead, by using dashes, we know that Cheryl, Blake, and Daniel are the three coworkers themselves.
4. F. As written, the sentence incorrectly employs dashes, offsetting “set the alarm” as a parenthetical when it is in fact an important part of the sentence. The comma and colon surrounding “remember” are appropriate, but we need commas to separate the individual items in the list. Changing the colon to a semicolon would create fragments that are not designated as part of a list. In a simple list like this, commas are sufficient.
5. A. Though the first part of this sentence (“When...factors.”) is an independent clause that could stand alone should a period separate it from the rest of the sentence, the rest of the sentence would not be able to stand alone, so the comma after “factors” must change. Velocity, acceleration, and the impact of gravity are all items in a series that describe the factors important to determining the path of a projectile. Thus, a colon is needed. And because the factors described are complicated and already separated by commas, semicolons are used to separate each factor rather than commas.
6. H. The text that follows the dash makes up a list of flavors. When a list contains three or more items, a comma must be used to separate each item. Currently, there are no commas, so “cherry orange” may be mistakenly read as one flavor rather than two separate flavors. So, a comma should be added after “cherry” and after “orange.”
7. D. Lists are used in sentences 1, 2, and 4. However, in sentence 4, the independent clause “Cooking is...pastimes” needs to be followed by a colon, since the items following it describe reasons why this is the case. As written, the sentence is a run on, since the comma between “pastimes” and “it” is inappropriate.
8. H. Lists are used in sentences 1, 3, and 4. However, the simple lists in sentences 1 and 3 do not require revision. In sentence 4, however, the sentence is very long and describes the opportunities mentioned in sentence 3. These are time with family, donating, or relaxing. However, the sentence describes “donating” in greater detail, offering examples like “time, money, or goods.” Instead, the ideas of spending time with loved ones, donating, and relaxing should be separated by semicolons.

## Parenthetical Expression & Nonrestrictive Clauses

1. A. The phrase “that was docked at the end of the pier” is restrictive, as without it the sentence is unclear (To which boat are residents referring? Where is the boat?) and the meaning lost. In general, a comma does not precede the word “that,” as the phrase that follows “that” generally restricts or

limits the thing that it modifies. In such a case, the clause is restrictive and does not need to be punctuated with commas, parentheses, or dashes.

2. H. The nonrestrictive clause “which is sometimes known as ‘mini golf’” should be punctuated with commas before it and after it. Generally, the word “which,” when used to precede an explanation or provide more information about something already mentioned (in this case, telling us another name for “miniature golf”) should be preceded by a comma. The clause should generally also end with a comma (when included as an interruption within a sentence). In this case, reading the sentence aloud can help determine where commas are necessary. The nonrestrictive clause only serves to provide more information about what “miniature golf” is, and doesn’t change the meaning of the sentence (that the game is popular with children and adults).
3. B. The phrase “who...offenders” is a nonrestrictive clause. Without it, we still know that Judge Lemon surprised the courtroom with an uncharacteristic action (her leniency is the opposite of “throwing the book”). The nonrestrictive clause simply tells more about Judge Lemon. Without it, we still know that she is usually more strict, since the courtroom was surprised by her leniency.
4. F. The extra information provided within the parentheses is not necessary to the basic meaning of the sentence – it simply provides more detail. Without it, the sentence still needs a comma to separate the dependent clause (“Unable...walls”) from the independent clause (“The client...coin”). Thus, the comma should go after “warming”). Changing the parentheses to commas does not solve the problem of adding a comma between the dependent and independent clauses, and actually confuses the sentence further with too many commas. We should not offset “which...walls” because this is not a restrictive clause. In this case, “which” is used as a determiner, not a pronoun, and describes a choice, not additional information about something else.
5. C. The phrase, “that were the most popular” is a nonrestrictive clause because it cannot be removed from the sentence without changing the meaning of the sentence. The information that the snacks are the “most popular” is needed to explain why they “were the first ones to run out.” Nonrestrictive clauses are not offset by commas and often begin with the word “that” (as does the one in this sentence). Therefore, both the comma before “that” and the comma after “popular” should be removed.
6. E. Nonrestrictive clauses and parenthetical expressions add richness and detail to writing. However, it must be properly offset with punctuation. Sentence 2 uses commas to properly offset “giving...family,” which describes more about “abruptly.” Sentence 4 is missing a comma between “trunk” and “which.” “Which...*Mayflower*” provides nonessential information to the sentence. The family was still amazed at the antique trunk in good condition. The additional information only adds the richness and detail to the sentence. The word “that” always signifies a restrictive clause, and should never be preceded by a comma.
7. A. In sentence 1, the clause “Frank was under the mistaken impression that the food on display was free” can stand alone as a sentence. By introducing additional information about why, we learn that Frank was unfamiliar with the local culture. This is a parenthetical expression, and needs to be offset with punctuation. There is one dash already at the beginning of the expression, but another one needs to be included at the end.
8. G. The expressions offset by dashes in sentence 2 and 4 are done so appropriately, since they provide additional, clarifying information to the sentence, but do not change the sentences’ fundamental meanings. However, in sentence 3, the clause “that confuse...most” is restrictive; it is necessary to the meaning of the sentence. Without it, we don’t know what questions the sentence is referring to. So, no dash is necessary.



## Unnecessary Punctuation

1. D. It is important to understand the rules for commas. In this sentence, there is an independent clause (“Cheetahs are one of nature’s most effective hunters”) and a dependent clause (“possessing both stealth and speed”). The latter cannot stand alone, and needs to be separated from the independent clause with a comma. Without it, there would be a run-on sentence. It is not appropriate to separate “cheetahs” from the rest of the sentence, because as written, there is no independent clause. The fact that cheetahs possess two qualities (stealth and speed) does not need to be separated by a comma, since it is a simple description.
2. F. The phrase “over the roar of the crowd” describes when Curtis yelled something. It cannot stand alone, and therefore must be separated from the rest of the sentence with a comma. However, no comma is needed to separate “roar” from “of,” since it is the roar of the crowd that Curtis is yelling over.
3. A. There are no commas required in this sentence. The sentence is long and complicated, but does not require additional punctuation beyond the period at the end. The sentence is not just about investment, investment in science, or investment in medical advancement, individually. Instead, it is about “investment in scientific and medical advancement.” The rest of the sentence describes this, saying that experts on all sides of the political spectrum agree on this statement. Adding extra commas results in a confusing sentence – on what sides are the experts on? What does the political spectrum have to do with anything?
4. E. There are no commas required in this sentence. The sentence is long and complicated, but does not require additional punctuation beyond the period at the end. There is no reason to separate “research” from what that research is. The sentence is about research studying the impact of news stations on the elderly. That impact is nervousness and anxiety, so no commas are necessary.
5. A. The comma following “work” unnecessarily interrupts the sentence. The text before that comma, “After a long day at work,” describes the text that follows, “at the office.” Therefore, this comma should be deleted from the sentence.
6. F. Punctuation in each sentence is used correctly except in sentence 2. “hard-to-reach” is punctuated correctly since it is a compound adjective that describes “shelf.” However, no hyphen should be included between the compound adjective and the noun itself. Notice in sentences 1 and 3 that the parenthetical expressions “who often...personality” and “during which...wanted” (respectively) are properly offset by dashes.
7. D. The last sentence should be written: “It is difficult to feel the awe – the sense of smallness – that the real world inspires without setting foot outside.” The parenthetical description of “awe” (sense of smallness) must be offset by punctuation, and there is no need for a comma after “difficult” because the clause is “it is difficult to feel the awe.”
8. G. The sentences in this paragraph are very complex, so it is important to identify the different parts of each sentence. In sentence 3, an extra comma after “philosophers” is unnecessary. The core sentence is “many of these philosophers agree upon the purpose of human existence.” The “in addition” at the beginning is a conjunctive adverb, and requires a comma after it. The phrase “which is...self” is dependent on the core sentence, and cannot stand alone. Therefore, it requires a comma before it to connect it to the rest of the sentence.

## Passage-Based/Editing Skills

### Choosing a Chooser

1. C. Sentence 3 discusses only the “power to rule over people,” not the distribution of money. It emphasizes that the people are the ones with the power to rule themselves. It would not support this idea to say that some democracies are like aristocracies (“a few well-established families”), which



are described in sentence 2. Neither would it make sense to say that only the rich have the power to influence the lawmaking process. Instead, the fact that the word itself derives from a Greek word meaning “rule of the commoners” provides the best support.

2. F. Sentences 5 and 6 describe how each type of democracy works. “Former” refers to “direct democracy,” and “latter” refers to “representative democracy.” The first choice combines both sentences, but rewrites it with a meaning that is the opposite of the original. The second choice properly uses the word “unlike” to draw a comparison between the two ideas. The third choice uses the phrase “despite that” while the fourth choice uses the word “because”; neither of these is appropriate given the fact that there are two very different ways of operating a democracy being described, and one does not happen as a result of the other.
3. D. Sentence 8 describes how people don’t have the time to learn about all of the details when voting on new laws or regulations, as would be the case in direct democracies. Instead, they’re busy, and elect others in a representative democracy. To support this idea, the author could include the last choice, since it reinforces this idea and provides examples of how people are busy. The other choices describe the opposite of the idea put forth in sentence 8, describing how people could “eventually learn” or would “rather serve” as elected officials.
4. H. Sentence 10 describes how people elect officials to make decisions for them. This is not an example of sentence 9 (sentence 9 itself is an example of why people choose to elect officials), as “to illustrate” would suggest. There are no similarities between sentences 9 and 10, so “likewise” would not be appropriate. Instead, sentence 9 provides an example of why sentence 10 is true, so the best phrase is “with this in mind.” “Regardless” would be used to signify a contrast between ideas, instead of the cause/effect relationship of “with this in mind.”
5. A. Sentence 14 attempts to describe how bigger groups have more power and could do things that do not benefit smaller groups. This is neatly summed up in the first choice. The other choices include vague phrases (“make things happen”) or wordy/redundant construction (“that group”; “each of these”).
6. E. The subject (“representative democracy”) is singular, so the verb “to be” must also be singular in the form of “has” (“have” is plural). “Benefits” is plural, as is “outweigh.” The dependent clause “though...challenges” must be separated from the independent clause “its benefits...drawbacks.” “Its” shows proper possession, while “it’s” is a contraction for “it is” that is inappropriate in this case.
7. B. Sentence 12 mentions qualities of a “perfect” elected representative. This is not the focus of the passage. Instead, the passage is concerned with describing the benefits of a representative democracy only. The other choices mention key examples of or summarize this.

#### Sweet But Deadly

1. C. The sentence describes how there are “a lot of different things said,” which is vague by itself (said where? By whom?). If something is “hard to know” or difficult to determine if it is true or false, then it is confusing. The first and second choices do not capture the idea that what is stated is often confusing. The last choice uses a vague pronoun “they.”
2. E. Sentences 2 and 3 describe specific instances where nutrition can be confusing, which is first put forth in sentence 1. Sentence 4 reconfirms the idea put forth in sentence 1, so the appropriate transition is “as a result,” which conveys the idea that the sentence is a conclusion, or effect of what was described previously. It doesn’t make sense to use contrasting transitions (“conversely” and “nonetheless”). Neither does it make sense to say “to illustrate,” since the examples were provided in sentences 2 and 3, not in sentence 4.
3. B. Sentence 8 discusses how the body uses carbohydrates. Sentence 9 describes carbohydrates as “critical.” Therefore, we can infer a sentence inserted between the two should discuss carbohydrate’s

importance, and give examples of how the body uses carbohydrates. The first choice does not address either of these points, instead conflicting with sentence 9. The third choice describes a theory about why some people crave refined sugar; this too does not refer to the ideas in sentences 8 and 9.

4. E. The point of sentence 11 is to explain how sugar leads to a quick burst of energy. Sentence 12 describes how the energy doesn't last. An immediate rush of energy doesn't necessarily mean that it is lasting, so the best way to connect the two ideas is to show a contrast or qualification. Thus, the word "but" can connect the two adjectives that describe the "burst of energy": "immediate" and "fleeting." It's not the drinking or eating of refined sugars that doesn't last long, but the energy that comes from that action.
5. B. The purpose of the paragraph, including the preceding sentence, is to describe the immediate negative effects of eating refined sugars: that they make blood sugar levels rise, and make people feel tired. This is echoed in the second choice, while the other choices mention off-topic ideas (as in the last choice) or make statements that contradict the main idea of the paragraph.
6. G. It may appear that the verb "make" should agree with "foods," but the subject of the sentence is actually "eating," which is a gerund (verb ending in "-ing" that functions like a noun). This is tricky, but if we remove the information "foods with...often," we are left with "Eating make[s] people feel too full..."
7. D. The passage is focused with describing why refined sugars are bad for health, not with describing other things that people should be eating. The other choices describe important parts of the author's main idea.

#### A Lasting Monument

1. D. The correct choice combines the sentences using the word "and." The choice that combines the sentences with a comma is a run-on sentence. The choices that combine the sentences with "but" and "yet" are incorrect because these ideas are consistent with each other, not in opposition to each other. Therefore "and" is the best conjunction to use.
2. E. The correct choice changes "has begun" to "began," as the verb needs to just be in the simple past tense here. "Has begun" represents the present perfect which is used to describe an action that began in the past and continues into the present. Since the sentence makes clear this was an action started in the past and completed in the past, the past tense is necessary. The future tense is also not appropriate. Adding a comma after "approximately" is incorrect because the entire amount of time was "approximately fifteen years." Changing "was completed" to "has been completed" is not correct because this verb also needs to be in the past tense, not the present perfect.
3. D. Sentence 10 is true because of Sentence 9. Thus, the correct answer is "As a result," meaning "for that reason" or "consequently." The other words do not convey the correct relationship. "Although" and would make this sentence into a fragment and are therefore incorrect. "Still" implies that sentence 10 should be true in spite of what was said in sentence 9. "Furthermore" would mean sentence 10 provides additional information to sentence 9, but does not convey the cause-effect relationship between these two sentences.
4. E. This choice is the most concise version of the sentence without losing any pertinent information. The other choices all further complicate an already unnecessarily wordy sentence, or include redundant information.
5. A. This choice is related to the idea of tourists visiting the Parthenon, which is the topic of sentence 16 and also mentioned at the beginning of sentence 17. The other choices describe other aspects of the Parthenon which do not relate directly to the fact that it is now primarily a tourist attraction.
6. E. The verb "are" needs to be changed to a singular present tense verb because this sentence has a singular subject. While the sentence starts with the word "there," the subject of this sentence actually

is found after the verb. This is the case for sentences that begin with “there” or “here,” in which one needs to look after the form of the verb “to be” to find the subject. In this case, the subject is the word “much.” Changing the verb to “were” (plural, past) is not grammatically correct, since the verb must agree in number with the word “much.” Changes to “much” or “to be” are not necessary, as these parts of the sentence are correct as they are written.

7. B. The correct sentence provides unrelated information about the Ancient Persians whereas the rest of the passage is about the Ancient Greeks and the Parthenon. The other choices discuss key ideas that are central to the Parthenon and how it has changed over time.

#### Building Tomorrow

1. A. Sentence 1 describes how infrastructure is “taken for granted.” Sentence 2 describes how without infrastructure, society would be “unable to function.” It is not because of sentence 1 that sentence 2 happens, as “hence” would suggest. The two are not equivalent, as “likewise” would suggest. Sentence 2 is not an example of sentence 1. Instead, sentence 2 shows a contrast to sentence 1 in that what is taken for granted is actually very important.
2. G. The paragraph containing sentence 9 discusses how investment can create jobs. The only statistic that includes information about jobs is choice C. The other choices describe other statistics that are related to transportation infrastructure, but do not have to do specifically with job creation.
3. B. The subject of the sentence is “investments” (“in the local community” is a prepositional phrase that tells us more about the type of “investments”), which is plural. The verb “begins” is singular (think “he begins to...”) whereas the verb “begin” is plural (think “they begin to...”). All of the commas are appropriately included in the sentence.
4. H. The paragraph describes how spending on infrastructure leads to positive effects for the “broader economy.” The preceding sentences describe how this happens. There’s no evidence to suggest that this phenomenon can be seen in other types of investment, that it leads to lower taxes, or that one type of job is better than another type.
5. A. The subject of the sentence is “expense,” which is singular. Therefore, the verb must also be singular. In this case, the form of the verb “to be” is “are,” which is plural. We must change this to “is.” Be careful not to confuse the subject and verb with other parts of the sentence. “According to...(NEC)” is an adverbial phrase that tells more about the rest of the sentence. Similarly, “second-highest” is a compound adjective describing “expense.” “[that] Americans have” also describes “expense.”
6. F. The last paragraph describes how investing in infrastructure can help save money and time. The preceding sentences describe how there is an additional cost associated with driving on roads in need of repairs. So, it makes logical sense that sentences 18 and 19 should describe how this could translate to savings. Only choice B describes the hypothetical situation where people could save more money and do more with their time.
7. D. The passage is not primarily focused on describing the expenses that Americans have. It mentions transportation expense only in context of the fact that infrastructure development could help lower the cost of transportation. The fact that housing and shelter costs are the highest cost is irrelevant to the main idea of the paragraph, let alone the passage.

#### Hammer, Feather, Vacuum

1. B. The sentence describes something that happens after experiments are proven true or false. The word “becomes” is written in the present tense, so “cease” should also to be written in the present tense as “ceases.” “Weather” is the word to describe the short-term climate of a place. Deleting all of the commas results in a run-on sentence, where adding a semicolon after “theory” results in a fragment.



2. E. The sentence seeks simply to provide an example of how a theory became a fact, or to transition from the previous paragraph to the current paragraph. There is no need to write “think about,” since “take” is a shorter and more direct imperative. Similarly, “the thing that” is better referred to as “the phenomenon,” since a “thing” could be any number of things. The other choices use more words to describe less clear ideas – the opposite of precise writing.
3. D. There is a distinction between when people believed in a theory, and why they do. This is confused in the other choices. To say “while” observations support the reason why people believe in the theory is incorrect, since the reason and the theory are related. This is the same by using the word “though” to join the ideas in the sentence.
4. E. Sentence 16 describes what “conventional wisdom” would have suggested would happen. This is the opposite of what Galileo found according to sentence 17. Therefore, only a transition that highlights this difference should be added to the beginning of sentence 17. “As a result” and “accordingly” show a cause and effect relationship; it was not “because of” the conventional wisdom that Galileo found the opposite to have happened. Neither is there a concession about the relationship between these ideas, as “undoubtedly” suggests.
5. B. There is only one chamber, and it is the chamber’s floor that the hammer and feather are suspended above. Therefore, the proper way to show possession for “chamber” would be to write “chamber’s”.
6. H. Sentence 23 describes how the result confirmed Galileo’s theory. His theory had nothing to do with stating that air resistance is stronger than gravity. It also said that it was not heavier things that fall faster than lighter ones. Instead, it simply said that gravity acts on everything equally, which is implied by the fact that the hammer and feather landed on the ground simultaneously without the influence of air resistance.
7. C. Sentence 12 describes nothing about Galileo’s theory, nor anything about how experiments help confirm theories into facts, but instead talks about how Galileo was punished for his work. This is not the focus of the rest of the passage, unlike the other sentences, which detail or transition between ideas of how experiments influence theories and facts.

#### Plastic, Plastic, Everywhere

1. C. Sentence 1 describes how people believe that so much plastic is recycled, but sentence 2 describes how very little is actually recycled. This shows a contrast, so we must use a transition like “despite” to show this contrast. “Because” shows a cause and effect relationship, which is not valid in this case. “In addition” means that the following sentence should be another related point, with a similar idea, which is clearly not the case in this case. “After all” is used to signal a conclusion of some kind; again, this is not appropriate, since the two sentences present contrasting information.
2. G. When multiple adjectives are used to describe a single noun, we must put commas between those adjectives. In this case, “swirling” is an adjective, as is “continent-sized.” Both of these describe “vortex.” To change tense (from “is” to “was”) would be incorrect, since the rest of the sentence paragraph is written in the present tense. The adjective “continent-sized” is a compound adjective, and must be hyphenated. To change “coasts” to “coast” is incorrect, since we are talking about two different coasts (California and Japan).
3. D. The sentence fails to agree in number. The first part of the sentence (“Some...chemicals”) can stand alone as an independent clause, and describes “toxic chemicals,” which is plural. The second part of the sentence describes more about “toxic chemicals” (that they make their way up the entire food chain). The problem is that “makes” is singular (think “he makes”), as is “its” (which is the possessive form of “it,” referring to a singular object). The plural form of these words is “make” and “their.” “Leech” is a noun, referring to the animal, or a verb, referring to the habitual reliance on something.



4. E. Sentence 11 describes how chemicals travel up the food chain. Sentence 12 describes the food chain as a situation where smaller animals are consumed by bigger ones. The two are related logically, so the word “as” is the only conjunction that makes sense. “Thus” suggests that because chemicals build up in the food chain, animals are consumed by others. The opposite is suggested by “despite” and “and yet.”
5. B. The main purpose of the paragraph is to describe how chemicals travel up the food chain and affect people. Only the second choice describes this as happening after presenting the reader with information about the food chain and how chemicals can travel up the food chain. This is the opposite of the statements made in the third and last choice. The first choice is illogical given the information presented in the rest of the paragraph.
6. H. We must be clear that we are referring to “the amount of plastic waste” and “the amount of plastic used,” which is different than referring to “plastic waste” and “plastic used.” We can do many things to “plastic waste” – we can burn it, eat it, etc., but we cannot do those things to “the amount of plastic waste.” “Minimize and shrink plastic waste” is confusing, since the sentence and paragraph conveys the idea of generating less plastic waste, not “shrinking plastic waste” (which could be construed as literally making plastic waste smaller in size). Saying “reduce, minimize, and decrease the amount of” is redundant.
7. A. Sentence 3 describes the process of recycling. While this may be interesting, it is only tangentially related to the main idea of the passage, which is about the prevalence and danger of plastic, and what can be done about it. It has little to do with the sentence that follows it, and is not by itself a reason for the information provided in sentence 2. The other sentences provide important examples or statements that are central the author’s main ideas.

#### Scary Money

1. B. The sentence seeks to explain and define what ROI means. The first choice simply gives us the definition of profit and investment, but not ROI. The third choice uses redundant phrases like “an acronym” and “for short.” The last choice also uses redundant phrases like “in businesses” and “businessmen” and “made by businesses” or “invested into businesses”. Only the second choice defines ROI in the most economical way.
2. H. Sentence 5 discusses how horror movies are inexpensive to make compared with other movies. Only this choice discusses the cost of making movies. The other choices do not continue this train of thought, instead diverting to different topics or contradicting the preceding sentence entirely.
3. D. “Energizing” describes how something gives energy to something else. “Energized” is the feeling of being supplied with energy. The commas are necessary to separate the different parts of the sentence, including the prepositional phrase “in the face of danger.”
4. E. The subject of the sentence is “movies,” which is plural. Therefore, the verbs that apply to “movies” must agree. In this case, the verb “triggers” is singular (“he triggers”) and should instead be plural (“they trigger”). The phrase “Research suggests that” tells us more about the core of the sentence, which is “Horror movies trigger the body [to produce...].”
5. C. The main idea of the two sentences is that excitement and giddiness can also happen “at the same time” as fear and dread.” The other choices do not properly capture this nuance.
6. F. This sentence summarizes what horror movies do to the minds of its viewers, which is described throughout the passage. “After all” is a concluding transition, which can also show a contrast with the preceding sentence. This makes sense, given that the preceding sentence discusses what horror movies do not need to do. “Besides” tells us that there should be more, new information (there is not), “certainly” adds emphasis which is not the case here, and “consequently” shows how something the result of something else (in this case, it is not because of what horror movies do not do that causes what they do).

7. C. Sentence 17 describes what action/adventure movies do to keep audiences interested. This is not the focus of the passage, which instead describes how and why horror movies make a lot of money without spending much money. The other sentences are important transitions or main ideas of the passage.

#### The Gambler's Fallacy

1. C. The nonrestrictive clause "which is colored green" describes more about the slot that isn't colored red or black. When a clause is nonrestrictive, and begins with "which," it requires a comma before it. The other choices make the sentence incorrect, changing the tense and number of the verb "to be." We know the clause is nonrestrictive because the sentence still makes sense without it.
2. H. To ensure that the comparison between nouns is appropriate, we must add the words in the answer choice. The sentence describes how the chance of the ball falling into each colored slot is nearly fifty-percent. As written, however, the sentence compares "the number of times the ball falls into red slots" with "black slots" themselves. The other choices change the tense or number of the verb inappropriately.
3. B. Sentence 11 tells us that the ball should fall into the red slots and black slots a roughly equal number of times. Sentence 12 describes how, over a period of time, the ball fell into only black-colored slots. "Afterward" describes a period of time, which does not make sense given the context. "Similarly" doesn't make sense, since the two sentences describe different things. Neither does "hence."
4. F. The preceding sentence describes how there should have been more times the ball landed in green and red slots. This is because the chance of a ball falling into a black slot is only about fifty-percent. The other choices contradict the information we are given in the preceding sentence or earlier in the passage.
5. A. Both sentences describe what happens when people fall prey to The Gambler's Fallacy. If a conjunction is to be used, then it should connect the two ideas together. The conjunctions "or" and "neither" do not; neither does "but not." The first choice describes giving up one set up beliefs for another set.
6. H. The first choice omits the fact that people forget the odds of something happening. The sentence is not about forgetting, as the second choice suggests, but is about ignoring, or falling victim to, the bias. The third choice uses vague language (who is "some," and what does "end up" mean?).
7. B. The passage does not delve into the different games that are offered at the casino. Instead, it is focused on using roulette as an example of The Gambler's Fallacy. The other choices represent important supporting or main ideas of the passage.

#### State of Confusion

1. C. The clause "China ceded Taiwan to Japan" is an independent clause, meaning it can stand alone. The phrase that comes before it, "Following...century," tells us more about the action of ceding Taiwan. Specifically, it tells us when it took place. It cannot stand alone as a sentence, and so therefore needs to be joined with the independent clause with a comma.
2. E. Both "accordingly" and "therefore" show a cause and effect relationship. However, it is not because of the preceding sentence, where Japan agreed to return Taiwan to China, that causes the current sentence, where Taiwan's return is complicated. Instead, it is everything else after sentence 7 that describes why it was complicated. This shows a contrast, so "yet" is most appropriate.
3. B. Sentence 9 is primarily concerned with describing how the nationalists and communists stopped fighting. Sentence 10 describes why. The first choice loses this nuance. The third and last use incorrect conjunctions like "but" and "neither/nor." Instead, it is possible to combine the sentence by saying that the two adversaries stopped fighting "[in order] to" repel the Japanese.

4. G. The sentence could end at “arose.” However, “which...Taiwan” is additional information that explains more about the question mentioned in the preceding sentence. It should not be separated from the rest of the sentence by a semicolon. Instead, we can use a colon to signify that there is a particular piece of information that needs to be introduced.
5. D. The first choice, though shorter than the other choices, provides nonsensical information (that the ROC has “falsely run” an “illegitimate government”? Does this mean the government is actually legitimate?). Similarly, in the second choice, we see redundancy in “illegitimately done” and “false government.” The third choice is verbose, saying “tells everyone who will listen” and uses vague words (“its own founding”). The last choice succinctly uses “argues” (instead of “tells people”), “illegitimate government” (instead of “false and fake”) and conveys the intended meaning of the sentence.
6. H. Sentence 22 describes Taiwan’s point of view. The last choice supports the ROC’s point of view, while the second choice directly counters it. The first and third choices are neutral.
7. D. Sentence 17 describes how Taiwan is a popular tourist destination. This is irrelevant, as the rest of the passage is concerned with describing the dispute between the ROC and PRC in terms of historical context.

#### Bad Risks

1. A. The two sentences convey different ideas. Sentence 3 describes how there was a “superficial” cause – or something that appears to be so on the surface. Sentence 4 describes how there was another reason but that this reason was “at its core.” The idea of something superficial is the opposite of something that is deep and meaningful. Therefore, the best combination of the sentences highlights this with the word “though.” “And” and “similarly” express congruency of ideas, where “still” conveys an idea that something is the case notwithstanding something else.
2. F. We must be careful not to delete too much of the sentence when editing for precision. Sentence 8 describes how loans were given to almost everyone even if they could not repay. Choice A fails to capture the fact about the inability to repay the loan – an important part of the original sentence. Choices C and D use roundabout ways of saying things. For example, it is implied that the loans are from banks or other financial institutions, so it is unnecessary to mention that the loans are from a bank. Similarly, “if it was determined that they would have a hard time repaying it” is a longer and less direct way of saying that the loans were given “regardless of their ability to repay.” Similarly, we do not need to know that it is the inability to repay a loan, since the fact that it is a loan is understood.
3. A. In this sentence, there is nothing wrong with offsetting the parenthetical expression “including those...it” with dashes. It would also be correct to offset the expression with commas, though doing so does not correct for anything wrong with the sentence. Instead, we have a problem with subject-verb agreement. The subject “any business” is treated as a singular subject, so the verb to be must reflect that number. “Is” is singular, where “are” is plural (think “he is” versus “they are”). We must keep the comma after “after all” since it is a connecting phrase which helps us to understand what follows it in relation to what precedes it. We cannot change “lent” to “lends” because the other verb in the sentence (“asked”) is already written in the past tense.
4. H. “So” indicates the appropriate cause/effect relationship between sentence 19, which describes an idea generally, and sentence 20, which describes specifically how that idea applies to the topic being discussed. “Yet” and “otherwise” are used to show a contrast, and “besides” is used to show something additional.
5. C. The other choices describe situations where managers and those responsible for the crisis are held “answerable” or “accountable” for their actions. Another way of describing this is to say that they were “punished” for their actions and forced to make things right with the rest of the country. This

runs contrary to the main thesis of the passage, and indeed, the paragraph, which describes how the people responsible “were able to avoid responsibility for the negative consequences of their risk-taking behavior.”

6. G. Sentence 12 describes how people who had lost their homes found new ones. The passage focuses on how the crisis was caused, and why it happened. It does not focus primarily on what happened to people as a result of the crisis.
7. C. The passage never discusses whether or not the crisis could have been avoided. Though the passage does mention how borrowing money to buy a house contributed to the crisis, it stops short of claiming that it is “unwise” or to be “avoided.” Instead, the passage is focused primarily on describing how risk taking and a lack of accountability enabled the financial crisis.

## Reading Comprehension

### Nonfiction

#### Befriend the Sandman or Beware

1. C. The passage is primarily concerned with explaining why sleep is important and how a lack of sleep is dangerous for drivers. Choices A and B describe aspects of sleep’s importance, but omit the information provided in the rest of the passage about how sleep affects driving. The author does not advocate for the reading of headlines and newspapers, only using an example cited from a newspaper to support his explanation. The author does not explain what, precisely, sleep is, or how it works.
2. H. The passage does not mention sleep’s impact on people’s immune systems. The 2<sup>nd</sup> paragraph describes all of the other choices.
3. B. Use of the article supports the author’s main idea, connecting the idea of sleep’s importance and the dangers associated with driving while fatigued. The inclusion of the examples from the article precedes mention of the ruling by the Federal Motor Carrier Safety Administration. There is no evidence to suggest that the two incidents described in the article occurred as a result of losing an hour or two of sleep. The article is included to show how the main idea is relevant to everyday life. While sleep is important to cognition, the examples provided by the article would more likely support the idea that sleep is important to responding to physical stimuli (i.e. driving).
4. G. The questions address the reader directly and are a stylistic way of writing to capture a reader’s attention. The topic of the questions is related to the main idea of the passage; that is, the importance of sleep and the dangers of driving while tired. The passage does not address objections or refute commonly held beliefs about sleep. While the author writes that some aspects of sleep are not well understood, the questions posed at the beginning of the passage do not relate to what about sleep is or is not understood.
5. C. According to the last paragraph, critics (i.e. those people who do not agree with the Federal Motor Carrier Safety Administration’s ruling) often cite that the hours are “still too long,” and compare those hours with average workweek hours (40-50 hours long). The passage does not support the idea that truckers must work at least 11 hours per day or that the 100,000 car accidents are all due to tired truckers.
6. E. The last paragraph states that the Federal Motor Carrier Safety Administration ruling took place in 2012 and reduced the upper limit of hours truckers can drive from 82 to 70.
7. B. The passage never describes the diversity of people’s sleeping habits and how that factors into driving safely. However, the passage does describe how sleep is not completely understood, but that it is still very important to traffic safety. Lack of sleep is singled out as the most important factor that contributes to traffic accidents; the passage doesn’t dive into any other factors that are as important.



8. G. Paragraph 5 describes the government's new laws aimed at helping reduce the number of traffic accidents caused by fatigue. This can be viewed as a solution to the problem of driving with a lack of sleep. The information about the magnitude of the problem was stated in paragraph 4. The figures in paragraph 5 describe more about how the changes can help reduce the magnitude of the problem.
9. C. The table presents data that shows the number of hours driven with no break and the increased chances of an accident. The risk increases more per 0.5 hours, showing that not every half hour of driving is equally "dangerous." This table doesn't show the number of people killed by drivers, or the ideal amount of sleep or driving per day.

#### Jack the Dripper

1. A. The passage follows Pollock from the beginning to the end of his life. There is only one point of view offered (the narrator's). There is no problem stated, nor is there an argument being advanced. Facts are stated simply in a chronological order.
2. G. The passage describes Jackson Pollock's life and work and describes him as a famous artist. While some parts of the passage discuss the art industry (Guggenheim's patronage of Pollock), other artists (Benton), and even the Works Progress Administration, they are all done so in context of describing Pollock's life in biographical form.
3. D. The third paragraph tells us that Benton's influence on Pollock was significant, and states that "many art historians believe that Pollock's artistic departure from tradition derives from Benton's strong sense of independence." The other answer choices are unsupported by the passage, as there is no indication that any of the other choices happened.
4. H. The Works Progress Administration employed many artists to work in the Federal Art Project during the Great Depression. We know this from the fourth paragraph. There is no evidence offered to support the other answer choices.
5. C. The sixth paragraph tells us more about how Pollock's drip technique earned "critical acclaim" and was "iconic." "Jack the Dripper" was a name given to him by *Time* magazine because of his drip painting. The passage does not indicate that these works were created for Guggenheim, nor does it mention that all the works were donated. We know that Pollock's most celebrated works happened while he was still alive based on paragraph six.
6. E. The passage never describes why Pollock stopped using drip style, only that it stopped "as suddenly as it began." There's no mention of negative feedback, and there is the opposite of a lack of interest.
7. D. We can infer from the third and fourth paragraphs that working as an artist during the Great Depression was difficult. The first sentence of the fourth paragraph tells us that Pollock "barely" got by until Guggenheim became a "paying patron." This phrasing lets us know that "keeping his head above water" has to do with money. We can infer the meaning from context clues. The fact that the art was inspired by water, that he had a paying job or had previously been in trouble with the law, are unsupported.
8. F. The author begins the passage by saying that Pollock's works are among the most influential and famous in the world. This, coupled with the statements about his success in paragraphs 6 and 7, suggest that buying his works might be well worth the expense. There's no mention of Pollock being reclusive. Since the WPA helped Pollock survive, and the author has a positive opinion on Pollock, we can infer that the author would be in support of government funded art projects. Pollock's drip technique was developed as a result of many factors, but we cannot make inferences about its development.
9. A. Part of his non-traditional upbringing was that he was raised in many different places and had exposure to many different experiences. The only statement that supports this idea is the first. The rest do not deal with his upbringing.

## The Lost City

1. C. Indiana Jones is a fictional character that would be more likely to be familiar to readers than Hiram Bingham III. The author uses this reference to engage the readers, and introduce Bingham, who is the main focus of the piece. Indiana Jones is only mentioned later in reference to Bingham, and the passage overall is not about Jones.
2. H. The passage focuses on how Bingham discovered Machu Picchu. Though Bingham's contributions as an academic are mentioned, they are background details rather than the central idea.
3. C. Paragraph 6 describes how Bingham saw the ruins of Choqquequirau on his first visit to Peru in 1909, and how "he believed differently" from the commonly held assumption that it was the last capital. His theory then leads him to organize the second expedition in 1911. He did not find Machu Picchu on his first visit; nor was his second visit a return to Choqquequirau. There is no mention of him doing research after returning from Cuzco; it was his experience of Choqquequirau, and not studies from America, that inspired him to return.
4. G. Paragraph 5 shows how Bingham's interest in Latin American history was consistent through his various academic positions. There is no mention of his childhood, financial resources, or other expeditioners in this paragraph.
5. C. Paragraphs 7 to 9 how Bingham meets a local man, Melchor Arteaga, who leads him directly to Machu Picchu. We can infer that without local help, in an unfamiliar terrain, Bingham would have been unlikely to find the ruins. There is no mention of dangerous obstacles, or of Bingham knowing the way himself prior to reaching Peru.
6. F. From the beginning of the story of how Bingham finds Machu Picchu, the author is comparing Bingham to Indiana Jones. The playful tone of this sentence mirrors the thrilling adventure-story style that Indiana Jones stories are told in.
7. A. In paragraph 3, the author calls Hiram Bingham "one of those lucky few" for coming "close to living out the fictional life of Jones." At no point in the passage does the author describe Bingham's discovery in a negative light.
8. F. Although academics are not necessarily the first thing that people think of when they hear the name Indiana Jones, this quote starts with "Like Jones, Bingham was..." and therefore highlights a commonality. The quote from paragraph 2 does not explicitly mention Bingham, and the quote from paragraph 11 discusses a difference, not a commonality.

## A Labor of Love

1. C. The passage describes two scientists – Mendel and Galileo – whose work was rejected during their lifetimes, but later found to be essential foundations for modern science. Though much of the passage focuses on Galileo, he is being used as an example to support the central idea, and the passage as a whole is not solely focused on his conflicts with the Roman Inquisition. None of the examples describe scientists dying for their beliefs; Galileo was only sentenced to house arrest.
2. E. The word "hail" means "celebrate," and "father of genetics" shows how important Mendel's work is now considered to be. After this sentence, the paragraph goes on to detail Mendel's ideas and how "his findings were initially rejected by his peers" (paragraph 2). As such, this sentence poses a contrast to the reaction that Mendel received during his peers. While it does show how modern scientists view him, the question is asking about structure and not simply meaning.
3. B. In paragraph 3, the author explains that "In Lorini's point of view, Galileo was reinterpreting the Bible, an activity that was prohibited by the Roman Catholic Church's Council of Trent," and that is the reason why the Church forbade his studies. There is no mention of Galileo's personal faith, and paragraph 4 goes on to show that he did in fact ask for permission to publish his work.
4. F. The word "persecution" means "ill-treatment," and the statement that "mere rejection, however unfortunate, pales in comparison to persecution" (paragraph 3) emphasizes how severe, or unpleasant, "persecution" feels. The examples of how the Inquisition treated Galileo (house arrest, being ordered to stop his studies) are more focused on how badly he was treated, rather than how

difficult it was for him to spread his ideas. In fact, the author does not mention whether Galileo prioritized spreading his findings, so much as his perseverance in conducting his research despite opposition.

5. A. In the sentence “Galileo obeyed – at least superficially” (paragraph 4), “superficially” or “only in appearance” describe Galileo’s obedience to the commands of the Church. The passage then goes on to detail how he wrote a book on his studies; the Church’s negative reaction to its contents implies that he did not modify his ideas to please them, despite following technicalities by asking for permission to publish. There is no mention of Galileo’s emotional state about these events, or of his faith.
6. H. Earlier in the paragraph, the author states that “Galileo did not live to see his work truly appreciated,” implying that Galileo’s work deserved appreciation, and that the author did not agree with the way the Inquisition treated him. This answer option is the best match, since it declares that “Galileo had been wrongfully condemned.”
7. C. The author makes a distinction between the “mere rejection” that Mendel received, versus the “prosecution” that Galileo faced, so they are different examples of ways in which scientists were historically underappreciated. Mendel worked on genetics and Galileo on solar systems, so they did not work in similar disciplines.
8. H. Paragraph 6 is about how Galileo’s worth as a scientist was eventually recognized many years after his death, which shows a change of mind from how he was treated in his present day. “Emphasizing how badly scientists were treated during their lifetimes” was discussed at other points in the passage, and not in paragraph 6.

#### Cellular Solutions

1. B. The passage as a whole is arguing in favor of stem cell researching, citing its scientific and medical benefits (paragraph 4-5). Though it discusses how stem cell research was once controversial, that is in the past (when the cells were taken from embryos), and scientists have already figured out a way around that issue, by using adult stem cells (paragraph 3).
2. F. The word “applications” means “uses.” In the context of this sentence, is it referring to the beneficial uses of stem cells. Although in other contexts “applications” can also refer to paperwork, that meaning does not apply here.
3. B. Paragraph 4 discusses how brain cells were “previously irreplaceable” before the discovery of stem cells, which implies that they do not naturally regenerate when damaged. Hence the example of a person with Alzheimer’s: without stem cell treatment, the disease cannot be reversed.
4. H. Although the passage discusses many uses for stem cells, it does not claim that stem cells are guaranteed to cure all medical conditions. The research is still ongoing, and many of the proposed uses are not fully developed yet.
5. C. In paragraph 2, the passage describes how scientists figured out how to isolate human stem cells in 1998. The year 1981 refers to when they first discovered how to isolate stem cells in small mammals.
6. G. Since paragraph 2 describes how complicated it was for scientists to “isolate,” or separate, specialized stem cells, we can infer that the embryos contained more than just that type – in other words, a mix of unspecialized and specialized cells. The embryos cannot contain only unspecialized cells, since that would defeat the purpose of searching for stem cells in embryos.

#### That’s Not Music!

1. B. The author is challenging readers to think more deeply about what makes music. The passage later contrasts the views of an average person (paragraph 8) with music theory (paragraph 2), and is mostly about how complex the melodies we take for granted actually are. The questions are not part of the author’s personal exploration of music theory.



2. G. The passage focuses on the most important element that makes music: melody. Stylistic changes in music and the composition process over time are mentioned but neither are the main focus of the passage.
3. A. The importance of melody in qualifying sounds as music is the central claim of the passage, and this quote supports that idea. Though the statement came from the student of a great composer, neither their ideological differences nor the idea that only great composers are concerned with melody are likely purposes for including the quote.
4. F. The idea that random notes in any order do not necessarily result in something nice to listen to suggests that there is a deliberate design process that must take place to create what can be called a melody. The quote from paragraph 1 hints at this idea, but pointing out a difference between music and noise does not necessarily imply that that difference arises from design.
5. B. In paragraph 3, the author asks the reader to consider what it is about melody that makes music pleasing to the ear. Paragraph 4 provides an answer to this question by discussing the technical aspects of a good melody.
6. G. The author discusses repeated exposure in paragraph 6, stating that repeatedly listening to a song can change the way people respond to it. This is not guaranteed, as the author uses the word “can”.
7. B. In paragraph 5, the author states that different people have different reactions to the same melodies due to variations in brain chemistry. This implies that it would be difficult to create one melody which many different individuals would like. However, the author is not implying that brain chemistry should be changed to guarantee a positive reaction.
8. G. The main point of the passage is that all good melodies share the same qualities that make them musically pleasing to listen to. This quote illustrates that these qualities have not changed throughout history, even though musical styles may have changed.
9. C. The author discusses the elements of memorable music that have stood the test of time, so he would likely agree that the length of time a melody is remembered and enjoyed by future generations would be a way to determine its quality. Though it is a skill to create a melody that is universally liked by all, the author would not say that this is impossible, given the examples of *Thriller* and *Für Elise*.

### Bridging the Gap

1. B. Paragraphs 2-5 show how bridges have become more structurally and technologically advanced with time. The central idea is not simply that there are different types of bridges, but that bridge designs grew more complex and ambitious over time.
2. H. The author discusses in paragraph 4 that suspension bridges solved the problem of weight and distance that arch bridges could not. It also discusses how some of the oldest bridges in history are arch bridges, so there is no evidence that they tend to crumble. Even though suspension bridges are aesthetically beautiful and challenging to build, these are not the reasons given for why people transitioned to using them.
3. D. This sentence establishes the idea that early humans built bridges from simple logs, which is a big contrast from the complex suspension bridges detailed later in the passage. It does not show the *recent* history of bridges, nor is the passage an overview of the historical innovators of bridge design. The quote is also not claiming the Nature was an “innovator” of bridges, since that would imply sentient intention.
4. H. Paragraph 3 states the central idea of the whole passage: “As technology advanced over the ages, people developed new ideas for building bridges. Each development allowed people to span ever greater distances.” There is no argument or counterargument in paragraphs 4-5, and paragraph 7 is about bridges in modernity.



5. B. Paragraph 4 explicitly states the types of scientific principles at work in building a bridge. The other choices describe bridges, but not how science was necessary to the development of bridges.
6. G. Paragraph 5 emphasizes the role of engineers in bridge construction, and how difficult their work is, from the many different factors they must take into account, to the dire consequences of their miscalculations. The paragraph does not discuss multiple famous engineers, or the impact of natural disasters on bridges.
7. B. A “feat” is an accomplishment that was difficult to achieve. By using the word “feat” to describe the engineering process behind the Brooklyn Bridge, an example of a modern bridge, the author implies that it is an impressive construction. Though “feat” implies something difficult occurred, there is no reference in the text to the difficulty of traveling over these bridges. Nor is there any mention of how big the cables are, or the heroism displayed during the Quebec Bridge collapse.
8. G. The table supports the ideas outlined in the passage by suggesting that bridges became longer through human history, since it gives the lengths of bridges from 1310 B.C. to 1937 A.D., and the lengths increase. This supports the claim in the passage, that longer bridges were possible due to improvements in technology and engineering. The table does not highlight the different types of bridges. This chart alone does not reveal that the Quebec bridge is the longest of its kind nor how sound the technology of the oldest bridges was.

### Pop, Pop!

1. D. Referencing Michael Jackson, an iconic pop figure, as well as providing statistical details about his sales history, helps establish the author’s credibility as an expert on pop music history. The entire premise of the first paragraph is to establish how success was established *prior* to social media, in the era of Billboard charts. Paragraph 1 also claims that pop music of that era did not have one universal style or sound.
2. E. The major claim of this excerpt is that pop music was historically characterized by diversity of musical styles, and is defined as an all-encompassing category based on popularity. Later on, the passage goes on to lament the lack of diversity in pop music now, so diversity does not still characterize pop music.
3. D. The first paragraph introduces pop music and its history. The third paragraph discusses how the musical qualities of pop music is now different in the present. The third paragraph beckons a question – “But what makes pop music so popular, anyway?” – that bridges these two larger discussions in the passage. This question is presented not to analyze or criticize the digitalization of pop music, but rather to dive deeper into how the music changed as a result of technological advances.
4. F. In the past, the success of pop music was predicated on album sales statistics and Billboard chart rankings. But, as paragraph 5 states, “Forget Billboard charts – today we need look no further than social media and online media outlets to determine the popularity of a singer or their songs.” Mass appeal still determines the success of pop music, as evidenced by the importance of online followings. Radio play is not specifically mentioned for either the past or present.
5. A. Paragraphs 4 and 5 argue that shortened attention spans, caused by digital changes, have forced many musical artists to rely on multiple hooks, which help maintain the attention of listeners. People have not requested these hooks, and there is no indication that they are easier to write.
6. G. The word “formula” refers to the widespread uniformity of music and formulaic composition of hooks, which the author openly laments later in the passage. The author is not impressed by the work of the songwriters, and does not describe them as “scientific.” Though the passage attributes part of this problem to listener tastes, which have been changed by technology, there is no direct blame for audiences.

7. C. The reference to Barack Obama emphasizes that some pop musicians have a greater social media following than the president at the time, which is a testament to their widespread impact. It does not help support arguments about the political influence or sway of pop artists, or the decline of political activism.
8. E. The numbers in the “Instagram Follower” column do not match the ranking order of the Billboard charts (especially in the case of Taylor Swift, whose social media following would rank her higher than the Billboard charts do). The social media followings and album sales do not match, so not all best-selling artists have the greatest social media impact.
9. B. Paragraph 6 highlights how, due to technology, pop stars are now able to generate profits through more than just album sales. With the internet, they are able to sell different products directly to fans, and are making more money than stars in the time of Michael Jackson. Though the examples may show how these stars are multi-talented and business savvy, that does not tie in to the central idea of the passage, which is about how technology changed the pop music industry.
10. F. The final question, “After all, 15 billion views can’t be wrong – right?” incorporates a hyphen in order to create an explicit of hesitation or pause. There is an inherent contradiction that is being conveyed through the juxtaposition of the words wrong and right, which are separated by this explicit hesitation. This creates a sense of ambiguity. The author is not encouraging further research or highlighting the inability to explain pop music. Instead, the author is suggesting that the line between wrong and right when it comes to the current direction of pop music may be ambiguous or blurred.

### The Legacy of Impressionism

1. D. The passage is primarily concerned with describing the wide-reaching effects Impressionism had on history and other musical genres. While Impressionism can be linked “to the revolutionary sounds of rock and roll in the 1960s,” it can also be linked to other artistic movements and musical styles, such as American jazz, so that is a detail rather than the main idea. The passage does not give an overview of all of music history for thousands of years, and the use of timbre is another detail.
2. F. The author states that music impacted other fields, but also that it drew inspiration from fields like visual arts. He/she never describes Impressionism as a “strictly separate” or “superior” art form. Instead, the entire article focuses on how Impressionism became more powerful because of its collaborative capacities.
3. D. The sentence from paragraph 4 captures the idea that Impressionist music drew inspiration from visual art, which is reinforced by the quote from paragraph 6, which highlights how Impressionists “experiment with the relationships...between sounds, colors, and feelings.” Diversity of sound, innovation, and change in popularity are all other ideas about Impressionism, and are not addressed by this particular sentence.
4. H. The third paragraph is mostly about how the “main objective of the Impressionist era was to paint emotional pictures through unique musical arrangements.” They did this by using a technique known as timbre, which the excerpt describes as a means to “stimulate the visual senses of the audience.” Therefore, it is an example of a technique. While timbre is utilized to paint emotional pictures, this is metaphorical language, and timbre has nothing to do with actual sculptors or painters.
5. C. Impressionist musicians wanted to capture visual beauty in their orchestral arrangements. When the passage describes “the calm evoked by light dancing on still waters” and “the tranquility of sunlight filtering through treetops,” it is directly referencing the impact visual beauty had on composing Impressionist music. These descriptions are not reflective of the settings in which Impressionists composed their music, but rather the mental images they tried to create for their audiences.

6. F. The word “manipulate” implies a task involving skill and delicacy. Its usage supports the idea that timbre was not easy to use, and that the composers used it skillfully. Though the words sound similar, there is no evidence to conclude the composers were manipulative – or sly, devious individuals.
7. B. Paragraphs 5 and 6 highlight the ways in which Impressionism influenced later musical styles and genres, such as American jazz and rock and roll. Paragraph 7 is the one that transitions into the idea that Impressionist concepts affected many people beyond the field of classical music.
8. H. The phrase “its beauty inevitably bled into other artistic disciplines” is used to describe the historical impact of Impressionist concepts blending with, or bleeding into, other art forms. Though other parts of the passage discuss how Impressionist music is influenced by visual art, this statement does not show evidence of that. It is also a general statement, and does not give an example of specifically how Impressionist music influenced other fields. The passage does not indicate anywhere that Impressionism lived in isolation from other disciplines.

#### A Different Lens

1. D. Paragraph 1 sets forth the claim that science fiction is more than just exotic settings and far-fetched technological premises. At its heart, it “tell[s] us more about ourselves.” Though the passage praises the relevance of science fiction, it does not compare it to other genres, so it cannot be said to be the most relevant genre. The main idea of paragraph 1 is that science-fiction is relevant to the everyday.
2. G. The central idea of the passage is that science fiction deals with human themes. This claim is supported by various examples (paragraphs 3-5), written by “the Big Three.” Though it briefly touches upon the plots of these books, that is in service to the central idea previously mentioned.
3. A. According to paragraph 3, “Lagath...is bathed in constant illumination by six suns.” The eclipse is an unanticipated and rare event, and not the reason why Lagath is unaccustomed to darkness.
4. F. The “Big Three” mentioned in paragraph 6 refers to Heinlein, Clarke and Asimov, who are all science fiction authors.
5. C. “Prophetic” means accurately predicted, so we can infer that the sentence implies that the issues described in the novel later became true in real life. As the book is about “technology and how it has many potentially negative effects on humanity,” that is most likely what has proven true since 1968. We cannot assume the inverse, that technology was beneficial to people before 1968.
6. E. As the Martian “explores the meaning of being human” in the book, we can infer that the Martian did not have a thorough understanding of human culture prior to contact. We cannot tell, from paragraph 4, whether the Martian was a powerful representative or provided humans with new technology. Lagath is the setting of a different novel.
7. D. The passage concludes by noting to prevailing popularity of the science fiction genre, in movies as well as books. That science fiction movies sell well is a supporting detail, and not the central idea of the paragraph.

#### Paved with Good Intentions

1. C. Paragraphs 2-4 describes that ecotourism is a type of tourism that attempts to support conservation, but is actually quite limited in making the impact it claims to. Although the article discusses tourism for social media self-promotion in paragraph 1, this is the type of tourism that ecotourism tries to prevent. The passage does not claim that ecotourism is illegal nor that it generates donations for conservation non-profits (in fact, it criticizes ecotourism for not sharing sufficient profits with conservation causes).
2. H. “Skepticism” means thinking that something is too good to be true. The tone of the question casts doubts on the idea of ecotourism because its premises are not realized in practice (for example, foot traffic not being zero impact). Paragraph 2 establishes the positive intentions of ecotourism and



Paragraph 4 exposes that the reality of ecotourism, so paragraph 3 offers a transition to the more skeptical tone of paragraph 4. The tone is not neutral or ambivalent because the author makes a clear stance against ecotourism.

3. A. Even though the United Nations Food and Agriculture Organization claims that ecotourism is nature-based and sustainable, the author most likely does not agree with this perspective, as the passage focuses on the unintended negative impacts of ecotourism (domestication of wild animals, loss of resources due to development, spreading human diseases, etc). There is no mention of the relationship between ecotourism and the high cost of living.
4. G. In paragraph 5, Daniel Blumstein “says that ecotourism is unaware of its consequences.” The example of the lion dying at the hands of hunters, because he felt safe around tourists, shows an unintended consequence. Blumstein’s quote does not discuss marketing or how the profits are used.
5. B. Paragraph 8 uses Costa Rica as a specific example of ecotourism gone wrong, where ecotourism is harming wildlife through sheer numbers of tourists arriving. There are no scientific claims in the paragraph nor does the paragraph summarize the entire passage. Even though the paragraph gives the historical context of ecotourism in Costa Rica, it does not discuss the history of ecotourism at large, which is the main problem in the passage.
6. F. Paragraph 6 discusses the phenomenon of domestication where animals adapt to the presence of humans and are therefore more vulnerable to danger, as an example of an unintended consequence of ecotourism. The passage does not suggest that this effect is intentional, nor does it discuss hunting regulations. Though the word “domestication” can, in other contexts, refer to the process of wild animals becoming pets, that is not how it is being used in this passage.
7. A. Paragraph 8 discusses how the rising number of tourists in Costa Rica has led to increased development, which has negatively impacted the local wildlife. This chart gives statistical support for the claim that there have been a rising number of tourists in Costa Rica between 1995 and 2015. The chart does not specify where the profits from ecotourism are going.
8. H. The author most likely mentions the ecotourism “buzzwords” – like “eco” and “green” – because they draw tourists into thinking they are making a positive impact while it actually makes a negative one. The author does not imply that the buzzwords are about native wildlife. The question of enforcing ecotourism practices is mentioned in a different paragraph.

#### A Critical Point of View

1. B. Paragraph 6 states: “Nevertheless, the themes that Carter covered – consumption, identity, and unity – are issues that still resonate today.” The paragraph then goes on to cite several examples of how the issues he described manifest in modern times (how climate change has escalated, how over-spending led to economic downturns), so it is not accurate to say that the central idea is only about the problems during Carter’s presidency, or that his speech helped fix the issues he discussed.
2. F. In paragraph 2, the author points out how remarkable it was for Carter to discuss spiritual problems in his speech, being the first U.S. president since World War II to do so. The other answer options mention aspects of his speech that would likely have occurred in other presidential speeches as well, or are supporting details.
3. C. The second paragraph states that “Carter began his speech by reading excerpts from letters sent to him by concerned Americans.” These letters can be categorized as a form of direct communication. While Carter also referred to national trends and news reports, it was the letters by American citizens that inspired his speech, not the advice of his speechwriters or other sources.
4. E. Paragraph 3 detailed some of the concerning human behaviors that support Carter’s claim of an American spiritual crisis. His tone does not convey dislike of American citizens so much as worry for America’s future. The comparison of the problems of the past with those of today occurs in paragraph 6, not 3.



5. D. His words call the American public to action, explaining that the “crisis could not be solved by politicians alone.” This is not a reference to his “lack of personal accountability” or his “personal crisis of faith” but rather a focus on collective action to “focus on contributing positively to society.” In this paragraph, Carter’s words are not antagonistic toward the American public; he is not blaming them.
6. G. The author’s perspective on Carter’s speech is largely positive, and he only references the fact that others believe Carter was being apocalyptic, unfair, and hyperbolic in order to offer a balanced historical assessment of American responses to the speech. Overall, however, the author seems to think highly of Carter’s justified concerns.
7. B. The word “bridges” is used here to show how the present connects to the past, and how the issues that Carter described have their equivalents in current times (i.e. global warming, economic depressions). These connections are not difficult to find, according to the passage, but rather are quite relevant, even though a lot of time has passed.
8. H. To be “visionary” means “to have foresight.” The author argues that Carter’s speech was not only accurate, but also predictive of future issues, as evidenced by how there is still a “crisis of confidence” by the time of Obama’s presidency. This statement counters critics’ claims about the apocalyptic and hyperbolic nature of Carter’s speech. Carter’s background as a Christian is mentioned to explain the spiritual focus of his concerns, and they are cohesive rather than competitive elements.

### Linguistically Speaking

1. C. The passage describes the many different ways in which linguists study language—from individual sounds and collections of sounds (phonology and morphology) to the way meaning is constructed and used (syntax, semantics, pragmatics), to historical and psychological aspects of language. This discussion reveals the complexity and nuance involved in language. Though the passage mentions the intuitive ease with which native speakers use language (paragraph 6), that is a detail and not the central idea.
2. E. The phrase “for granted” is an idiom, meaning that something is assumed and/or unquestioned. So, the passage is saying that we do not question the fact that we can communicate with each other. But as the passage goes on to suggest, communication as viewed by linguists is a far more complex process than is commonly perceived. There is no mention in paragraph 1 of young children.
3. D. The purpose of paragraph 2 is to present examples of some of the aspects of language that are studied by linguists, and it is the first paragraph to define the topic of linguistics. Some questions are presented in the paragraph, but not all of them are answered by the passage. Paragraph 9 is the conclusion, and does not elaborate on paragraph 2.
4. G. The passage compares words to the building blocks of a language. In keeping with this metaphor, groups of words like phrases and sentences would be buildings and skyscrapers, since they are each made up of many individual components. Just as individual words make up a phrase or sentence, so too are buildings and skyscrapers made up of different individual pieces, and are therefore complex. There is no reference to construction workers.
5. C. Morphemes include root words, prefixes, and suffixes, and the paragraph discusses how the meaning of a multi-syllabic word can be formed by combining the meanings of its morphemes. Therefore, we can infer that the more morphemes a person knows, the easier it will be for that person to determine the meaning of unknown words.
6. H. Phonology refers to the sounds that make up words. The only one of the choices that refers to sound is the one that mentions the similarity in sound between “cheese” and “zebra.”

7. C. The passage begins by stating that we take for granted the way we are able to communicate with one another, but then discusses the complexity of language that lies beneath the simple fact of our communication. The question at the end of the last paragraph is rhetorical, and reinforces both the way communication seems simple to us and the reality of how complicated it really is.
8. E. The last sentence of paragraph 8 says, “By breaking down the process by which children acquire language, psycholinguists can gain broader insights into how people of all ages learn.” While it may be more difficult for adults to learn a new language, there is no reference to adults being uncooperative.

#### Plastic Addiction

1. A. The passage focuses on the massive amounts of plastic in seas and how it effects our marine ecosystems, so this answer option, which cites that we are “choking our...seas with 6.3 tons of...plastic waste,” is the best fit. In paragraph 6, the author closes the passage with “we just need to *want* to do it,” but this call to action is only mentioned in the conclusion, and not throughout the passage as a whole
2. F. The excerpt implies that the majority of the plastics in the ocean could have been recycled and therefore diverted. This demonstrates how it is inevitable that plastic will eventually end up in the ocean. This particular excerpt does not mention that “plastic pollution” is increasing.
3. B. By stating that “700 different species” are threatened by plastic, this option supports the idea that a “variety of marine life” are affected. Though one option mentions that “a third of the fish caught [...] housed microplastics,” and addresses the number of animals affected, it does not discuss the variety of species affected.
4. H. The author writes that the nanoplastics can be absorbed into “our own bodies.” The use of first person “our,” and the fact that many readers probably consume fish, show that the author is appealing to the reader’s self-interest. Although the excerpt may appeal to a reader’s empathy, it is not focused on the harm to the fish so much as the eventual impact on humans from consumption.
5. A. The complaint that it takes “environmentally friendly plastic” 450 years to decompose (paragraph 2) shows that the author does not want any kind of plastic, biodegradable or not, to be in the ocean. The passage also says only “seven percent” of plastic is recycled (paragraph 2), and “we have technology to pick up, dispose of, and recycle plastic” (paragraph 6), which shows that the author believes it is realistic and possible to eliminate plastic waste in the ocean. The author states that plastic is damaging to the planet, but never says that it should be completely banned, only that it should be cleaned up properly.
6. G. The final paragraph discusses systematic changes (more garbage trucks, big companies committing to recycling product waste, countries putting plastic bans into effect) rather than individual changes, so it is not a call to action for readers to personally attempt to solve the problem. Paragraph 6 also does not summarize “human factors that caused the overall conflict.”
7. D. The author says “we have the technology” to recycle better, if we “*want* to do it.” This paragraph both re-frames the problem as one of human choice, and offers a solution. The technologies mentioned are real, not hypothetical, and “no politicians are claiming that ocean pollution is a hoax” (paragraph 6).
8. G. The graphic shows a trend over time. The amount of plastic is given in the y-axis, and the year on the x-axis. We can see an explosive growth in the amount of plastic waste generated, and that only a small portion of this is being recycled. We do not have any evidence that laws are being passed or that plastic is ending up in the ocean.

#### Two Treatises

1. C. The author describes how Locke believed “the powers of government should be divided” (paragraph 3), so we can infer that he would also believe that no one person should hold too much

power. His resistance of the monarchy also supports this. Though he disliked the monarchy, there is no mention of how Locke felt about the Parliament.

2. G. The passage as a whole is focused on Locke's ideas about governance and how they influenced others. Though there is information about Locke's life, the passage does not mention Locke's childhood, and only discusses the aspects of Locke's life that are relevant to his philosophical idea.
3. D. In paragraph 3, the author quotes Locke advocating "for the right of the citizens to rebel," if the ruler is unfair and attempting to "compel...by force." Though paragraph 2 discusses Locke's idea that governments require the consent of their subjects to be ruled, he does not imply that government should take the form of a monarchy.
4. H. Paragraph 1 explicitly states that Cromwell passed away when Locke was studying at Oxford to become a doctor.
5. A. Since Locke's philosophy was in opposition to monarchies, most likely Charles II and James II would disagree with it.
6. F. Locke was an antagonist to Charles II, not a supporter. However, he did serve Lord Shaftesbury, who was an opponent of the monarchy on the Parliament. His father's experiences in the Parliamentary forces likely influenced his political views as well.
7. A. As paragraph 5 states, "Almost a century later, Locke's ideas engendered a new revolution." This shows how Locke's ideas remained influential long after his death and in lands far away from his own. Though America's separation from England is mentioned in this paragraph, that is a supporting detail and not the main point.

Dear Evan Hansen

1. B. The passage is specifically focused on Andrew's hard work and passion toward pursuing a theatrical career when he "competed in the National School Musical," "starred in school and community theater," learned musical instruments, and wrote music. Winning the high school musical theater contest is only the first part of the passage, which also goes on to discuss his childhood and his role of Evan Hansen in great detail. Being an outsider looking in describes the character of Evan Hansen, and not Andrew himself.
2. F. Winning a national competition is a great exhilaration, but then to obtain not just a part in a Broadway production but the lead—that is an astonishing feat at 16, and sounds more like events from a fiction story than reality. The story-like use of repetition in these opening sentences reinforce that feeling of surrealism. The events described are not common/typical.
3. C. Paragraph 2 describes Andrew's journey at the National High School Musical Theater Awards, and specifically tells the multiple challenges that Andrew had to overcome, which demonstrates the extent of his accomplishment when he does win the contest. Even though the passage summarizes Andrew's childhood, that happens later, in paragraph 5.
4. E. The sentence shows the moment that all of Andrew's prior preparation culminated in, when he is finally rewarded for his efforts by being noticed by industry professionals. Although we can infer that it took a great deal of skill and practice for Andrew to sing his medley, this particular sentence addresses the reward and not the process.
5. D. The paragraph educates the reader on the character Evan Hansen—a 17-year old that is always trying to belong—and, that the role is an "emotionally intense one," usually cast by actors in their 20s. This fact is a testimony to Andrew's talent, since he is much younger than all previously casted actors. Even though "adult men" had previously played Evan Hansen, the paragraph never indicates that the role should only be played by them, since Andrew, a teenager, was also selected.

6. E. Paragraph 8 first uses the term “win” then changes it to “earned,” which has the connotation of something you worked for, or deserve. Andrew won the competition due to his hard work and dedication, not luck. Even though Andrew will likely earn money while on Broadway, the word “earn” here is not being used to describe financial earnings.
7. D. The sentence says his current schedule is “topsy-turvy” and lists his weekly tasks, which illustrates his continued strong work ethic. The quote from paragraph 5 describes his work ethic prior to the win, and not after. And although the statement about giving “five performances” may seem difficult, it goes on to say that producers are wary of asking him to give eight, which does not indicate a strong work ethic so much as the idea that he is still young and needs to be protected.
8. G. The author’s upbeat tone about Andrew’s success in the opening of paragraph 2 and the illustration of Andrew’s passion for performing (paragraph 5) shows us the author believes one should follow his or her passion. Even though the passage demonstrates the need for determination, it never guarantees success.

#### Masterminds of the Sea

1. D. The passage details how intelligence has been categorized in mammals (paragraph 2) then it goes into how octopuses, a cephalopod, contradicts those norms (paragraph 3). While the topic is octopus intelligence, the central idea is specifically how octopuses demonstrate intelligence even though many of their traits do not match scientists’ conceptions for intelligent animals. Being “fun” animals is a supporting detail, not the central idea.
2. H. Paragraph 2 “highlights the three most common traits” that indicate high intelligence: large brains, long lives, and social skills. The paragraph does not illustrate the methodology (technical step by step process) scientists use to identify the most powerful brain.
3. B. In paragraph 2, the author talks about scientists’ observations about high intelligence animals. In paragraph 3, the author’s use of “on the other hand” conveys how octopuses are a contradiction to these ideas. “Breaking the mold” is a phrase that also generally means to be an exception. Even though the passage demonstrates their activity in containment (paragraph 7), these details reinforce an octopus’s intelligence and personality, and not a desire to escape. Also, while aggression may be seen in individual octopuses – a fact that the author mentions to support the idea that octopuses have different personalities – it does not characterize the species as a whole.
4. H. Paragraph 4 illustrates examples of octopuses adapting to situations in ways that demonstrate intelligence (ex: tool use, problem solving) in their daily lives. Even though the passage explains how “experts viewed and recorded intelligence of the octopus,” the octopus mentioned were in the sea and not aquariums.



5. A. In paragraph 5, the author explains how an octopus shows the ability to make decisions, which is an “even more sophisticated,” or challenging, type of intelligence. The paragraph then goes on to detail specific experiments where octopuses showed clear decision-making skills.
6. F. In paragraph 7, the author lists various octopuses’ personalities (naughty, aggressive, emotional), and supports this claim with examples of ways octopuses demonstrated preferences among caregivers. Though we may correlate those behaviors with young children, the passage does not.
7. C. In paragraph 6, the author says an octopus is not just interested in food but wants “to do something interesting.” Throwing a jar repeatedly does not satisfy any basic survival needs (food, breeding, etc), but rather shows a need for mental stimulation. The example of tool use shows intelligence generally, but not that octopuses specifically need mental stimuli.
8. G. The word “road” here is used to describe metaphorical pathways to intelligence. To say that there are “different roads to...intelligence” means that there are different ways animals can develop high intelligence, as evidenced by the octopus, which is an outlier to conventional expectations. “Road” is not being used as a literal reference to travel.
9. B. The graphic compares the average total number of neurons among various mammals and the lone cephalopod, the octopus. It shows that the octopus has neurons that number as many as a cat, which is a mammal, showing that it is more intelligent than people may expect for a cephalopod. It does not prove that this is the only indicator of intelligence, or show that the nervous systems are similar. There is no mention of cat behaviors, or how those might compare to octopus behaviors.

#### The Cost of Convenience

1. B. The passage describes the environmental problem caused by plastic bags, as well as how this problem arose: the industrial revolution led to people being able to cheaply produce many types of different things, including containers and bags. The tax is designed to take away the appeal of using these bags all the time, in order to reduce waste. A history of containers and an explanation for why plastic bags are bad for the environment are provided, though they are all offered to advance a greater point, and are not described in detail throughout the passage. There is no explanation offered as to why the environment is more important than convenience.
2. E. The first two paragraphs describe why carrying one’s own container “was a commonly accepted fact of life.” The reason for this, the passage states, is that the “containers were valuable.” There’s no indication that people only bought what they could carry in their hands (if anything, people only did that because they couldn’t carry more containers with them). We don’t know if people in the past valued convenience more or less than they do today.
3. D. The third paragraph highlights how, with the industrial revolution, suddenly things like bags became cheap to produce. Because of this, everyone could have a bag or “as many bags as they wanted.” This does not mean there is more money, just that things like containers cost less than they did before. The invention of plastic and taxes are not cited as reasons for why people valued convenience over the containers themselves.
4. E. In the second-to-last paragraph, various environmental effects are listed. Soil erosion is not among these, but the rest of the answer choices are.
5. C. The question comes at a point in the passage where the focus shifts from explaining how bags became ubiquitous to why that is a problem. The question goes on to be answered in the next paragraph and describes more than just theoretical examples of how people use plastic bags.
6. F. The passage never indicates that the money generated from the tax will be used to clean up the environment, nor does it suggest that people will stop using plastic bags completely. Instead, people can be expected to value the convenience less, thereby generating less waste. This will help the environment in turn.

7. B. The graphic shows the amount of plastic generated vs. the amount recovered or recycled. This supports the fact that the “vast majority” of plastic is “simply thrown away.”

## Fiction & Poetry

### “In Time”

1. B. The poem’s focus is on the changes to the tree over the course of a year. The poem ends with the cycle beginning again, showing that the tree will always be changing. The new life in the bud does not appear in an unexpected place; if anything, it is expected because spring always comes. The poem doesn’t represent spring as the best season, nor does it suggest beauty can be found anywhere, since winter is not depicted as beautiful.
2. E. The tree’s branches are described as “fingers” that are “reaching out to greet” the speaker and “gently brushing the window pane.” These depict the tree as kind and friendly, meaning it would be a welcome sight to the speaker. The personification describes the tree in a lifelike way; it does not make the tree a representation for an actual person.
3. C. The second stanza focuses on the spring, but it does not suggest that the tree looks most beautiful in this season. The speaker’s mood may decline when winter arrives, but there is no evidence that it is because the birds left. There is no evidence of people or animals causing harm to the tree. The second stanza shows that the speaker finds beauty unique to each season because in this stanza, all of the details the speaker admires are particular to the spring.
4. E. Carpet is soft and cozy, and it creates a quieter sound when stepped on. Therefore, this comparison of the flower petals to a carpet contributes to a hushed and comforting tone. The comparison doesn’t suggest any inconvenience, the texture of the tree isn’t mentioned, and the carpet is not a literal carpet in the speaker’s home.
5. D. The mention of whispers describes the sound of the leaves rustling; there is not actual whispering and therefore no presence of mysteriousness. The people depend on the tree, not the other way around. The tree is appreciated by the neighborhood during several seasons, not just summer. The figurative language shows a connection between the tree and the people, since the people are drawn to the sound of the trees rustling and depend on the tree for shade.
6. G. The comparison of the leaf colors changing to fire conveys a feeling of shock and awe, since fire is a very bright and shocking sight. In these lines, the speed at which the leaves fall is not mentioned, nor is the sound of the rustling leaves. There is also no mention of damage to the leaves.
7. B. The mention of the growing chill and the squirrels frantically trying to prepare for winter creates a sense of impending doom, which is coming in the form of winter. While squirrels may be cute animals, they are not depicted as sweet or endearing here. They are also not shown as being carefree or playful but instead as diligently working. The speaker does not express an attitude of agitation in these lines.
8. G. The description of the gray sky and the image of an “barren nest” contribute to a mood of dread and the speaker’s feelings of hopelessness. The branches are not described as actually moving in these lines. There is no evidence that the speaker thinks the birds will never return, nor that the tree is beautiful or unappreciated during winter.
9. A. The repetition of “no” is used to describe a lack of life each time it is used in this line, therefore stressing the lifelessness of wintertime. There is no reference to how long winter has dragged on. The speaker may not enjoy winter as much as the other seasons, but there is not enough evidence or a strong enough tone of distaste to suggest he hates winter. The mention of the petals and the leaves is used to show how empty the tree is, not to compare their colors.
10. F. The bud, which will eventually bloom into a flower, represents hope. In the lifelessness and hopelessness of winter, the new life of the bud is a sign that the good times of spring are going to

come again. The bud does not hint at the end of life, only the start of new life. There is no evidence that the tree is trying to communicate with the speaker. The bud does not represent futility, since that would suggest that the possibility of new life is impossible.

### “Revolutions”

1. C. The phrase “a gentle tug at my ocean’s heart” suggests a subtle yet profound connection between the speaker and the object of their affection. It implies a delicate, yet significant, influence that the object of affection has on the speaker, akin to a gentle gravitational pull. This imagery of a ‘gentle tug’ contrasts with the notions of overwhelming influence (D) or yearning (B). It also doesn’t imply the unpredictability or changeability (A) of the relationship, but rather a steady, quiet presence that profoundly affects the speaker.
2. H. The description of the ocean’s movements as a “dance of waves, a song of ebb and flow” emphasizes the harmonious relationship between the speaker and the influencing celestial body. This imagery suggests a rhythmic, coordinated interplay, akin to a dance or a song, between the speaker (presumably a celestial body like a moon or a planet) and the object of their affection (possibly a larger celestial body like a planet or a star). It evokes a sense of balance and beauty in the natural movements, rather than unpredictability (E), destruction (F), or chaos (G).
3. C. The indented stanza structure in the poem emphasizes the conversational tone, suggesting a dialogue or exchange between two entities. This structure effectively reinforces the sense of a back-and-forth interaction or ‘conversation’ in the cosmic setting described in the poem. It highlights the give and take nature of the relationship between the speaker and the object of their affection, mirroring the gravitational interplay and influence between celestial bodies. Options A, B, and D, while relevant to poetic structure, do not capture this conversational aspect as effectively.
4. G. In these lines, the “chain” metaphorically represents the binding force of gravity. This invisible yet powerful force holds celestial bodies in their respective orbits, similar to how relationships can have unseen yet significant influences. The metaphor underscores the gravitational pull as a symbol of connection and attachment, drawing parallels to how relationships, though not always visible, exert a profound influence. Options E, F, and H, while touching upon themes of force and connection, do not directly relate to gravity’s role as a fundamental force.
5. B. The line “In your absence my oceans lie still” implies a sense of longing and waiting in the speaker. This line suggests that the speaker, perhaps a celestial body like a moon, experiences a notable change when the object of their affection, possibly a planet or star, is not present. The stillness of the oceans in their absence indicates a deep connection and dependence on the gravitational or influential presence of the other entity. It conveys a sense of quiet anticipation and a pause in activity, awaiting the return of the influential force. Options A, C, and D offer different interpretations: A suggests complete dependence, C indicates a literal end of existence, and D implies indifference, none of which align as closely with the sentiment of longing and expectation expressed in the line.
6. H. Referring to the object of the speaker’s affection as “a jewel in my sky” in line 13 expresses its value and significance, highlighting its crucial role in the speaker’s world. This metaphor conveys a sense of preciousness and importance, indicating that the object of affection is treasured and held in high esteem by the speaker. The use of “jewel” implies something rare and beautiful, suggesting that the object of affection has a unique and special place in the speaker’s existence. Options E, F, and G, while related to qualities of a jewel, do not capture the overarching idea of value and significance in the context of the poem.
7. C. The recurring theme of dance enhances the poem’s meaning by introducing an element of romance and elegance. This metaphor infuses the celestial setting with a sense of beauty and grace,



akin to a harmonious dance among the stars. It contributes to the portrayal of the relationship between the speaker and the object of their affection as a delicate, intricate, and beautifully coordinated interaction. The imagery of a ballet suggests a fluid, synchronized movement, reflecting the natural, yet precise, orbits and gravitational interactions in space. Options A, B, and D, while engaging with the notion of movement, don't capture the romantic and elegant aspect brought forth by the ballet metaphor.

8. E. The correct answer is E. The phrase "changing face, yet constant in your sight" implies that despite any changes or phases the speaker goes through, they remain a steady and constant presence in the life of the object of their affection. This suggests a commitment to being a reliable and unwavering figure, regardless of life's changes. The phrase reflects a dedication to maintaining a consistent presence, even as external or personal circumstances evolve. Options F, G, and H offer alternative interpretations, but they do not align as closely with the idea of steadfastness and constancy in the face of change as E does.
9. C. The phrase "a conversation, an eternal embrace" suggests a permanent and unchanging bond between the speaker and the object of their affection. It evokes the idea of a connection that is enduring, constant, and everlasting, symbolized by the imagery of an ongoing conversation and an embrace that never ends. This depiction emphasizes the depth and permanence of their relationship, portraying it as timeless and unbreakable.
10. F. The poem conveys a sense of timelessness in the relationship by referencing the eternal nature of celestial movements. This metaphor suggests that the relationship, much like the predictable and unending orbits of celestial bodies, exists beyond the confines of time, enduring and unchanging. It implies a bond that is as enduring as the celestial phenomena, remaining constant through time. Options E, G, and H, while offering different perspectives on timelessness, do not directly connect with the celestial imagery and the sense of enduring constancy as effectively as F.

#### "Explorer Amongst the Stacks"

1. D. This phrase illustrates the explorer's perception of the place as both magical and serene. It evokes a sense of enchantment and awe, capturing the childlike wonder and imagination that turns the environment into a dreamlike realm. Option A, focusing on vastness, does not convey the magical quality. Option B, while mentioning the quiet atmosphere, does not capture the dreamlike and whimsical aspect. Option C's focus on fragility and transience does not align with the emphasis on wonder and magic in the phrase.
2. G. The description of "towering canyons of knowledge" suggests that the explorer is in a place similar to a library. The detail of the explorer tiptoeing shows that they are being careful and respectful of their surroundings. This imagery evokes a sense of grandeur and depth associated with vast accumulations of knowledge, similar to what one might find in a storied and historic library. Option E, indicating overwhelm, is not directly implied by the imagery of canyons or the tiptoeing. Option F's focus on complexity and obstacles reflects the tiptoeing, but does not reflect the grandeur and richness implied. Option H's suggestion of mystery and arcane nature, while intriguing, does not directly align with the imagery of towering canyons of knowledge.
3. B. The mention of "creatures long vanished" and "bones etched in ink" suggests that the explorer is discovering stories or information about prehistoric life and extinct species. This imagery is evocative of paleontology and the study of ancient creatures, likely through books or illustrations that depict these extinct beings. Option A's fantasy world concept, while creative, does not directly align with the mention of "bones etched in ink." Option C, focusing on impermanence and fragility, and Option D, suggesting that the explorer is actually time traveling, do not directly relate to the specific imagery of prehistoric life and fossils.



4. G. The line, “Here, science paints the poetry of the skies, revealing the sun’s dance with the rain,” illustrates the merging of scientific understanding with the beauty of nature. It suggests that scientific concepts, like the formation of rainbows, are not just mechanical explanations but poetic revelations of nature’s beauty. Option E focuses on the visual experience of light, option F on the vivid colors, and option H on the discovery through reading, but none of these capture the blend of science and poetic beauty as effectively as option G.
5. D. The presence of elves and fairies in these lines indicates a shift in the explorer’s journey into the realm of fantasy and folklore. This shift shows the explorer moving from exploring the realms of natural phenomena and ancient history into the imaginative and fantastical world of myth and legend. Option A’s focus on blending myth with nature, Option B’s unsubstantiated claim that this is the explorer’s favorite genre, and Option C’s suggestion of interest in ancient myths’ moral lessons, while relevant, do not capture the transition into the fantasy genre as directly as option D.
6. F. The use of personification in “where trees speak and rivers sing” brings the elements of nature to life and enhances the magical atmosphere of the forest. By attributing human-like qualities to the trees and rivers, the poet creates a vivid and enchanted setting, making the forest seem alive and dynamic. This personification deepens the reader’s engagement with the natural world, making it more fantastical and whimsical. Options E, G, and H, while relevant to literary analysis, do not directly address the animating and magical effect of personification.
7. D. This line, “Here, the past breathes, each duel and dynasty alive within the fortress of words,” reflects the idea that ancient history and its stories come to life in the reader’s imagination. It suggests that the historical narratives, depicted in books, become vivid and animate in the reader’s mind, as if history is ‘breathing’ and living again. Option A doesn’t describe history coming to life, merely serving as an introduction. Options B and C describe elements of history but do not encompass the comprehensive idea of the past becoming alive in the reader’s imagination as vividly as option D.
8. H. The phrase “the universe contained on every open page” illustrates the concept that each book one encounters is a gateway to a new and different world. This idea aligns with the theme of exploration and discovery in the poem, where each book offers an entry into a unique realm, whether it be historic, fantastic, scientific, or mythic. Options E, F, and G, while related to the theme of exploration, do not specifically focus on the transformative power of books as gateways to different worlds. Option E suggests being intimidated, which is not the primary sentiment conveyed. Option F implies access to all knowledge, which is broader than the specific idea of books as gateways. Option G, focusing on the boundless nature of surroundings, does not directly link to the transformative aspect of books.
9. A. The tone of the poem is whimsical and imaginative, effectively capturing the explorer’s sense of awe and adventure as she discovers various worlds through books. This tone reflects the wonder and excitement of exploration, as each new book brings a different realm to life. Options B, C, and D, while they describe aspects of the poem, do not capture the essences of wonder and creativity as vividly as option A. Option B’s nostalgic tone is more retrospective, option C’s contemplative tone is more reflective, and option D’s focus on learning and education (while an important theme of the poem) misses the overall tone. Remember: tone is the author’s point of view on the subject, not your own interpretation of the poem.
10. F. The culmination of the exploration metaphor in these lines suggests that just as the universe is vast and boundless, so too are the possibilities and adventures offered by literature. This metaphor emphasizes the limitless potential of reading and imagination, paralleling the exploration of books with the exploration of the universe. Options E, G, and H, while relevant to the theme of exploration, do not capture the sense of boundlessness and infinite possibilities as directly as F.

“Picture Perfect Waves”

1. D. The phrase suggests a sense of comfort and security aboard the speaker’s “private wooden haven,” indicating a personal and familiar space where the speaker feels at ease and sheltered. Options A and B, while relevant to a maritime setting, do not capture the intimate and secure ambiance implied by “haven.” Option C, focusing on isolation, suggests a more negative connotation than the warmth and safety conveyed by words like “private” and “haven.”
2. F. The description of “the ocean’s boundless blue” serves to emphasize the vastness and freedom of the marine environment. This phrase conveys a sense of limitless space and endless possibilities for the speaker and reflects the expansive and liberating nature of the sea. Options E, G, and H, while they might be relevant to an ocean setting, do not capture the primary emphasis on vastness and freedom. Option E suggests a sense of introspection, option G suggests a boring quality to the ocean, and option H focuses on danger, none of which are present in the quote.
3. D. The imagery of the “towering painted canvas” and “gold and green” creates a vivid, picturesque scene that enhances the nautical atmosphere of the poem by painting a poetic image of the boat’s sails. These images paint a colorful and lively picture of the boat’s environment, contributing to the overall sensory experience of the setting. Options A, B, and C, while relevant to a maritime setting, do not reflect the quoted images. While line 4 does mention salty gusts of wind flapping the gold and green, validated part of option A, the “towering painted canvas” does not relate to the weather or time of day.
4. E. The personification of the waves clapping and celebrating with the speaker suggests the speaker sees the ocean as lively and active, indicated a harmonious relationship between the two, but not necessarily between the speaker and any other element of the natural world. ‘This imagery creates a sense of the ocean being animate and engaging, contributing to the feeling of a joyful and shared experience with the natural environment. Option H, while hinting at a positive interaction, doesn’t capture the dynamic liveliness specific to the relationship between the speaker and the sea as directly as E.
5. D. This phrase implies a deep familiarity and connection with the boat, as indicated by the speaker’s bare feet memorizing the deck’s worn grooves. It suggests a sense of comfort and history with the vessel, reflecting a long-standing relationship or frequent experiences aboard the boat. Options A, B, and C, while relevant to a maritime setting, do not capture the intimate connection and familiarity implied by the phrase. Option A suggests discomfort, option B indicates unfamiliarity, and option C implies physical toll, none of which align with the sense of familiarity and ease conveyed by the phrase.
6. E. The description of the sky as “an endless dome of azure dreams” contributes to the dreamlike, expansive setting of the poem. This imagery evokes a sense of vastness and wonder, encapsulating the speaker’s awe and imagination as they behold the immense, open sky. Options F, G, and H do not accurately reflect the sense of expansiveness and dreamlike quality. Option F suggests confinement, which is contrary to the expansiveness implied. Option G focuses on the feeling of insignificance, and option H on weather changes, neither of which align with the awe and expansiveness conveyed in the description.
7. A. The phrase “a lulling rhythm, a dance with the tides” reflects a feeling of relaxation and harmony with the ocean’s movements. It suggests a comforting and rhythmic interaction with the sea, portraying a peaceful and soothing experience. The speaker appears to be in sync with the natural rhythms of the ocean, finding tranquility in the gentle swaying and the rhythmic motion of the tides. Options B, C, and D, while related to the sea experience, do not capture the serene and harmonious connection with the ocean. Option B suggests a nonexistent relationship to the speaker’s sleep patterns, option C implies boredom, and option D a desire for excitement, none of which align with the calm and contented feeling conveyed in the phrase “a lulling rhythm, a slow dance with the tides.”

8. H. The gulls in lines 15-16 act as a manifestation of the speaker's excited emotions while sailing. Their cries, described as a "wild symphony," contribute to the sensory richness of the setting and emphasize the speaker's sense of exhilaration in the nautical environment. Options E, F, and G, while relevant to a maritime setting, do not capture the role of the gulls as a mirror to the feelings of the speaker. Option E suggests a negative aspect, option F a sense of chaos, and option G a proximity to land, none of which align with the gulls' function as a symbol of the speaker's feelings of excitement and energy.
9. B. The transition back to reality in lines 23-24 suggests a reluctant return to real life from a vivid daydream. The bell ringing and the gaze moving from the image on the wall indicate the end of the imaginative journey and a return to the more mundane setting of the classroom. This shift highlights the contrast between the speaker's rich, imaginative experience and the reality of their physical environment. Options A, C, and D, while potentially relevant, do not capture the specific sense of reluctance and contrast between daydream and reality. Option A suggests a change in mood and perspective, and while the speaker's attitude changes, their perspective of the serenity of the sea remains the same. Option C suggests a dissatisfaction with the environment, and option D an inability to distract himself, neither of which is present in the poem.
10. F. The framing of the boat as "framed in simplicity" contrasts the simple, still image of the boat with the complex and vivid imagination of the speaker. It suggests that while the physical representation of the boat is straightforward and unadorned, the speaker's daydreams inspired by it are rich and elaborate—that the boat's simplicity is just an echo of the vivid dreams. This contrast highlights the power of imagination to transform a simple image into an expansive and adventurous mental journey. Options E, G, and H, while relevant, do not focus on the contrast between simplicity and complexity as directly as F. Option E emphasizes the difference between fantasy and reality, but neither the image or the daydream concerns reality. Option G concerns the artistic aspect of the image, and option H suggests the speaker's desires to move onto another daydream, but neither of these options are implied by the quote or capture the juxtaposition of simplicity and imaginative complexity as effectively as option F.

**"microscopic world"**

1. A. The imagery effectively conveys the complexity and dynamism of the microscopic world, suggesting that within a single drop of water, there exists a thriving universe teeming with life. This vivid portrayal emphasizes the depth and richness of the microscopic realm, often overlooked due to its size. Options B, C, and D, while offering different perspectives on the included lines, do not offer any perspective on the a central theme of "microscopic world" as directly as A. Option B focuses on the poet's perspective, option C on the contrast between visible and invisible, and option D on scientific advancements, none of which are the central focus of this imagery or the passage.
2. H. This phrase uses personification by attributing human characteristics, specifically dancing, to microscopic entities. This literary device brings the microorganisms to life, making their world more vivid and relatable to the reader. It enhances the imagery by portraying their movements as an elegant and lively dance. Option E suggests hyperbole (exaggeration), which is not the primary effect here. Option F would explicitly compare microorganisms to humans and mention something about relationships between them, which it does not. Option G is close, but the direct attribution of human qualities to microorganisms makes personification a more accurate description.
3. A. The poet's language in these lines enhances the poem by illustrating the rapid and dynamic nature of life at the microscopic level. The comparison to shooting stars emphasizes the speed and energy of the microorganisms, portraying their world as active and vibrant. This vivid imagery brings the microscopic world to life, highlighting its liveliness and dynamism. Options B, C, and D, while



reflecting aspects of celestial comparison and wonder, do not directly address the dynamic nature and energy of the microorganisms' movements as effectively as option A. Though option B is close, the focus of the lines is not on the scale or impressiveness of the microorganisms, but on their frequent and hurried movement.

4. F. This line captures the intricacy and beauty of the microscopic world by comparing the movements of microorganisms to a ballet. This metaphor suggests grace, elegance, and careful choreography, characteristics that reflect the poem's theme of the beauties and complexities of the micro world. Options E, G, and H, while relevant, do not focus as directly on the complicated movements and beauty of the microscopic world through such vivid imagery as option F.
5. D. These lines show the invisible forces that govern the movements of microorganisms, drawing attention to the natural, yet unseen, phenomena that drive life and movement within a drop of water. This contributes to the poem's meaning by emphasizing the complexity and mystery of the microscopic world. Options A, while highlighting aspects of beauty and comparison, do not focus on the mystery and complexity of the invisible forces driving these movements. Option B, about the limitations of human observation, is not directly addressed in these lines, which are more about revealing than concealing aspects of the natural world.
6. G. The word "waltz" is used to illustrate the structured and rhythmic patterns of the microorganisms' motion. This choice of word conveys a sense of elegance and order, reflecting the idea that the movements of these tiny organisms are intricately coordinated and harmonious, much like the steps in a waltz. Option E suggests chaos, which contradicts the orderly nature of a waltz. Option F implies a contrast that is not the focus of this line. Option H, while capturing the playful aspect, does not emphasize the structured and rhythmic nature of the motion as clearly as option G.
7. D. By writing the poem in lowercase, the poet mirrors the minuscule subject matter and suggests that even things which are microscopic can possess their own inherent beauty. If the words of the poem, which are written in lowercase, can be beautiful, so can the microscopic world. Though Options A, B, are valid uses of the lowercase in poetry, there is no evidence to suggest they have bearing in this poem. Option C implies that the poem concerns a physically large subject—this is completely untrue.
8. E. The imagery of discovering "a universe within a universe" through the microscope's eye is used to emphasize the vastness and complexity inherent in even the smallest elements of nature. This line suggests that within the microscopic world, there is an entire universe of complexity and detail that mirrors the larger universe, highlighting the depth and intricacy found at the micro level. Option F, suggesting limitations of scientific tools, and Option G, about the level of human understanding, do not capture the sense of wonder and discovery that the imagery conveys. Option H, while related to the concept of hidden layers, does not directly address the parallel between the micro and macro universes as clearly as option E, and suggests a definite future that is not discussed or implied anywhere in the poem.
9. C. The tone of the poem is thoughtful and amazed. It is characterized by a deep sense of wonder and respect for the intricacies and beauties of the microscopic world. The poet's use of descriptive language and vivid imagery conveys a tone of marvel and contemplation, inviting the reader to share in the sense of awe at the complexity and beauty of the micro world. Options A, B, and D, while representing possible tones, do not match the poem's contemplative and awe-struck nature. Option A is too analytical, option B too warning, and option D too playful to accurately reflect the poem's tone.
10. H. In the last two lines of the final stanza, the poet uses details to indicate that the microscopic world is an untapped source of inspiration and discovery. By stating, "a testament to life's intricacy and complexity, / a hidden world: alive and waiting," the poet emphasizes that this minute universe, often unnoticed, is rich with complex life and phenomena. It suggests that there is much to learn and



marvel at in this hidden realm, waiting to be explored and appreciated. Option E suggests urgency, option F focuses on a call to action, and option G hints at future scientific breakthroughs, but none of these options encapsulate the sense of a world ‘alive and waiting’ for exploration and appreciation as effectively as option H.

### “Field Trip”

1. B. This phrase sets the tone for the museum setting, emphasizing its quiet, contemplative atmosphere, which is typical of such environments. Options A, C, and D, while plausible, do not capture the primary function of the phrase as effectively as B. Option A’s suggestion of isolation and detachment is more interpretive and doesn’t directly align with the phrase’s description of the museum’s atmosphere. Option C, regarding the protective nature of the museum, is a valid point but is secondary to the primary emphasis on the atmosphere, implied by the word “hushed.” Option D does not make sense within the context of the poem.
2. G. This description emphasizes the meticulous and careful process involved in reconstructing the dinosaur’s skeleton, akin to piecing together a complex puzzle. This simile highlights the precision and attention to detail required in preserving the subject for the present. Option E, focusing on size and complexity, is relevant, but nothing suggests the dinosaur is beyond comprehension. Option F, while poetic, implies that the skeletal reconstruction indicates mysteries that will never be solved. Option H interprets the description as suggesting an unnatural creation, which diverges from the emphasis on the careful assembly of the puzzle-like bones.
3. B. The imagery of the dinosaur as a testament to nature’s raw creation portrays it as a symbol of untamed, primal forces. This contributes to the poem by emphasizing the power and grandeur of nature as seen in the dinosaur’s massive form. Option C, focusing on awe and respect, is relevant but secondary to the portrayal of nature’s raw power. Option A and D, while thoughtful, don’t directly align with the imagery of nature’s raw creation as vividly as Option B.
4. E. This phrase suggests that civilization is only a smudge in time compared to the time of the subject, contrasting the scale and nature of the dinosaur’s time with the comparatively small and industrial modern world creates a stark contrast between the natural, unspoiled world of the dinosaur’s era and the industrialized, modern world. It emphasizes the purity of the dinosaur’s habitat (untouched) compared to the “smudge” of modern civilization. Option F, while related, doesn’t directly address the contrast or time scale as does E. Options G and H are relevant but are not as directly connected to the temporal contrast between past and present as A; each either focuses on the past, the present, or the contrast, but E is the only one which focuses on all three.
5. B. The tone of the poem is wonderstruck and admiring, reflecting a deep sense of marvel and admiration for the majesty and grandeur of nature as exemplified by the dinosaur. This tone is evident in the vivid and respectful descriptions of the dinosaur and its ancient world. Options A, C, and D, while they could be applicable to different contexts, do not capture the primary tone of awe and reverence as effectively as B.
6. G. The phrase “worlds long lost” reflects the speaker’s awareness of the immense changes that have occurred over geological time, emphasizing the deep history and transformation of the Earth since the era of dinosaurs. This acknowledgment highlights the temporal distance between the present and the dinosaur’s time. Options E, F, and H, while interesting, do not directly connect to the specific idea of recognizing geological time changes as clearly as G.
7. A. These lines portray the subject as being dynamic and lively in the past and static and unmoving in the present. Though B, C, and D all refer to the passage of time, none of them present the liveliness of the dinosaur’s former life in contrast to its present quietness.
8. H. The word “emissary” suggests that the dinosaur acts as a representative or ambassador from the ancient world, offering a tangible connection to the modern world. This implies a role of bridging

time and conveying the essence of a long-gone era to present-day observers. While options E, F, and G present interesting ideas, they do not directly align with the concept of the dinosaur as an emissary or representative as clearly as H.

9. A. The capitalization of each line of the poem conveys a relationship between the visual qualities of the poem and the grandeur and majesty of the dinosaurs. By intentionally beginning each line with a capital letter, the poet indicates an importance of the subject, drawing attention to each line. This style decision mirrors the scale and magnificence of the dinosaur. Options B, C, and D are irrelevant to the subject matter of the poem.
10. H. The repetition of “silent vigil” in the poem emphasizes the enduring presence of the past as symbolized by the dinosaur, in the modern world. It underscores the idea that elements of the ancient world, though silent and static, continue to hold a significant presence and relevance in contemporary settings. While options E, F, and G offer interesting perspectives, they do not capture the primary focus on the enduring presence of the past as effectively as H.

#### “When You Are Old”

1. B. The act of reading is one of reflection – a person reads words on a page but processes them through his or her own experiences and knowledge. This is similar to reflecting on the past. We don’t know that the speaker is in retirement, and that this is what the metaphor represents. Instead, from context with the rest of the passage, we know that this is a time for reflection, not a time to literally read a book. There’s no mention of writing an autobiography, or sharing lessons learned from life with others.
2. F. The word “you” can be directed toward a specific person, or to a group of people. In this case, we don’t know who the speaker is addressing. This is deliberate on the speaker’s part, as a central idea of the passage, then, can apply to anyone. In this case, the speaker can be addressing herself directly, or can be telling someone (or some group of people) what to do. The speech is more of an internal dialogue that we are privileged to, as opposed to a conversation.
3. C. The glowing bars recall back to the earlier image of a fireplace. This is in the last stanza of the poem in which the speaker reflects on the loss of that true love. The glowing bars are the flames in the fireplace, which hasn’t been compared with youthful beauty and love before (those were described in the second stanza). It is in the first stanza where the speaker falls deep into her thoughts.
4. F. The “soft look” describes the look of the person’s eyes, and represents youth or innocence. This is echoed by the line “moments of glad grace,” as opposed to “sorrows of your changing face.” The former describes youthful grace (remember to read the lines before and after the ones in question), while the latter describes the pains of growing old (as does “old and grey and full of sleep”).
5. D. The second stanza describes how the speaker was beautiful, and had many loves, but these loves were “false or true.” This suggests that physical beauty can cause a superficial love, which is only skin deep, and may or may not have been true love. However, one person loved the speaker’s soul, and the “sorrows of your changing face.” This is true love – love that stays with a person through the ups and downs, no matter the outward beauty.
6. H. The fact that “Love” is capitalized is significant and intentional. The speaker is still very sad, and it is not thanks to love that she has recovered (she hasn’t), but because of it that she is sad. The specific person that was her love has not been identified, and here, the speaker does not speak directly to the concept of love, as apostrophe would suggest.

#### “Leisure”

1. C. The speaker addresses Leisure herself with a strong desire, wishing that she would bless the over-busy-world today with her presence. She may be frustrated by the lack of leisure in the present, but there is a desire and longing in her writing that extends beyond just frustration.

2. E. Apostrophe is the impassioned, direct address made by someone to an abstract idea or inanimate object (or an absent person). This creates a conversational tone where emotions are revealed. The speaker does not address a person, nor does this addressing of Leisure directly cause confusion or comic relief.
3. D. These lines tell us that nowadays, people are haunted by the reminder of things undone, and so they feel they have no time to enjoy life. The word heritage does mean customs and beliefs, but not in the sense of tradition. Instead, it means that the customs of daily life are too arduous and keep people too busy, leading to “shortening moments.”
4. H. After telling us that leisure is withheld from “this over-busy world,” the speaker tells us that leisure can still be found, but only in the “unhewn woods” – or nature. While this could be construed to mean that this is alone and without other people, we don’t know for sure whether or not the speaker means that leisure can only be found by oneself. Instead, we can take this to mean that the speaker believes leisure can be found away from society, in nature.
5. D. The “crime” referred to in line 11 is elaborated on in the lines that follow: “Not to have worshipped...the dreaming lapse of slow, unmeasured time.” We are being told to enjoy the passage to time for itself. Therefore, the answer that most would be like this “crime” would be to rush from one thing to another, not enjoying the time. All the other activities are contemplative or involve enjoying the passage of time.
6. G. When reading anything, we read commas as short pauses, and periods as stops. In the first 9 lines, there is no end-of-sentence punctuation. Indeed, the longest pause is a semicolon, which is not quite as “strong” of a stop as a period. The effect of this lack of punctuation is that the poem reads uninterruptedly. The many commas cause us to pause, and slow down – ideas reflected in the actual words and ideas of the passage.

### “Song”

1. A. The speaker consistently describes how he feels. In the first stanza, he describes the first time he saw her, and the effect it had on him. This effect gives us a glimpse into how he feels – that his spirit was a “deeper night” and that the only thing that had ever shone on “its shadows drear” was her light. The other choices describe his reactions, but don’t give an insight into his frame of mind.
2. H. All of these stanzas begin with a different time that the speaker sees the woman. The fact that the speaker counts these times is significant, almost obsessive. The stanzas describe how the woman has had positive effects on the speaker, making the speaker more excited, happy, and glad.
3. D. The simile compares the woman’s effect on thrilling the speaker’s heart to a “wild hand” (active hand) playing a stringed instrument (lyre). This is not meant to be taken literally. We don’t know that the woman is a musician. This is the opposite of being calmed by the woman.
4. G. The comparison to the woman as a “fadeless flower” suggests that in the speaker’s mind, the woman’s image is indelibly ingrained. Note the lines that precede these: “I saw her oft again—, each hour / Enhanced o’er me her conquering power”. The other lines describe her appearance or sound, as well as the ultimate effect the woman has on the speaker.
5. A. The 7<sup>th</sup> and 8<sup>th</sup> stanzas focus on describing the woman’s beauty in more detail. This echoes back to the first line, where the woman’s beauty is first mentioned. The other lines describe the effects the beauty has on the speaker.
6. G. The stanzas follow the pattern: AABAB, which repeats throughout the entire poem. Like this, the general idea of each stanza is the same in that they describe a quality of the woman and its effect on the speaker.
7. B. The fact that nothing the woman could say would be less beautiful than the sweet song of a bird suggests that the woman is perfect, to the speaker. Note the context: the stanza immediately



preceding this one describes how even her footsteps are “More beautiful than if a God / Had placed immortal foot-prints there”.

Adapted from *A Modern Tomboy*

1. A. From context, we can tell that the word is describing the amount of time in which the rugs can be rolled up.
2. H. Lucy turns red because she dislikes Rosamund’s suggestion, not because she was jealous of Rosamund’s ideas. There’s no evidence that Rosamund was humiliated, since shortly thereafter, she gives a “smile which seemed to denote power.” Instead, Rosamund has usurped (seized control over) Lucy’s party, and is giving orders even to Lucy, the apparent host.
3. B. The tone, which is clearly not a phrase meant to be taken literally, is positive. In context, we can tell that Phyllis had gone off to find permission, and just returned to find out that it has been granted. This means that she was likely animated, not direct (in which case she would not have used such a long phrase), particularly dignified, or impartial. She wanted to dance, otherwise, she would not have gone to get permission. The phrase is colloquial, not formal or dignified.
4. G. The expression “go to Hong Kong” is used to mean Lucy’s mother doesn’t care about the rugs. We can infer this because Lucy becomes enraged, as this was the opposite of what she expected, and now she has no reason not to let Rosamund dance.
5. C. We don’t know much about Rosamund and Lucy except for what is shown in this passage. However, we do know that at the center of the conflict in this passage is Rosamund’s ability to get her way: once with the dancing, which Lucy objects to, and another time with Lucy’s father, the professor, which Lucy also objects to. There are no allusions to the fact that Rosamund’s appearance, dancing, or conversations are the object of Lucy’s rage (though the latter is part of what makes Lucy angry toward the end of the passage).
6. G. Lucy clearly did not want Rosamund to move the furniture, since she reacts to the suggestion with widened eyes, which shows she is shocked. Knowing this and that her parents have everything placed just as they “wish it to be,” we can infer that the current arrangement is not set up for dancing as it is and that Rosamund believes her parents would not approve of changing the set up. We cannot infer that her parents would never approve of playing music and dancing because once they are asked, they give their permission for them to move the rugs and dance.
7. C. Lucy is boiling with rage at the fact that Rosamund has gotten her way the entire night. Lucy opposed the dancing, but Rosamund made it happen. Lucy didn’t want Rosamund to talk to her father, but she couldn’t stop Rosamund.
8. G. It appears that Phyllis had an idea that something like this would happen, because the secret looks that she gives to Lucy drive Lucy mad with rage.

Adapted from *The Jolliest School of All*

1. A. The issue in the passage is the fact that Lorna and Irene are friends, but that this friend is related to her father’s enemy. Thus, there is tension between family and friend.
2. E. Paragraph 9 tells us that Lorna’s father knows “his [Beverley’s] writing only too well,” all of which was prompted by the revelation that Lorna had been reading a book written by him.
3. B. Lorna likes Irene and describes this friendship in paragraph 11. This includes statements that Irene was kind, loving, accepting, and could be trusted. The feud between the fathers had not affected the girls (this is the first that Lorna has realized that her friend’s father is her father’s enemy).
4. H. The passage only shows us that Lorna and Irene were friends, never that Irene was the object of Lorna’s jealousy. The passage states that Irene was “the only one at school who had sympathized and understood her.” This shows that Lorna was clearly not popular, and in fact, had difficulties making friends.



5. B. The only sentence which describes Lorna's internal struggle are in this choice, where the narrator describes the realization as a "blow" that was "overwhelming." A drowning man battling the waters is as vivid an image of a struggle as one can imagine!
6. H. These questions are asked in the last paragraph of the excerpt. In this paragraph, the reader is shown how Lorna has confusion ("whirling eddies of her thoughts") about how to respond but ultimately concludes that forgiveness is the only solution ("Curses, like chickens, come home to roost").
7. A. The desired purpose of the passage is to help the reader to empathize with Lorna. We aren't told why David Beverley and Lorna's father are feuding. Instead, we are given a glimpse into Lorna's feelings, and thus, are asked to feel for her.
8. H. The understanding that revenge will only make things worse (the last line in the passage) prevents Lorna from following her father unconditionally.

Excerpt from *"Ride Proud, Rebel!"*

1. D. The author gives us clues as to what the "roan" is. It has a tail, which is not helpful since all of the choices have tails. However, we know that it is big enough to have a "rider" and that it is saddled (since it was unsaddled). The author also writes about how it is a "mount." Thus, it is most likely a horse.
2. H. The passage gives us many clues about Drew's experiences. The fact that there is an "economy of action" is one such clue. However, the bigger clue is from paragraph 7, which describes his "boyish roundness" as being melted only through "hard experience."
3. A. Reading the paragraph in context, we know that Drew is "remembering" and that he is recalling his friend, Sheldon, and how they built "it" (Boone's Fort), and how this was his only piece of property. He looks out onto the scene, but is looking vacantly, as he recalls the passage of time, and his memories.
4. G. We can infer that Drew is currently at Boone's Fort, and that because nobody knows about it (paragraph 6), it was a retreat that he had built with his friend, Sheldon.
5. A. We are not told what his present purpose is, but there is no evidence to suggest that he is there to simply enjoy Red Springs, or even to join up with people at Chickamauga. Instead, we can infer that Drew is working on some unknown task.
6. E. Since the sentence tells us that it was "gone in a flash" and left "no hint of softness," we can tell that Drew was faking the smile. His features are described as "gaunt" and "dark" – two very negative descriptors.
7. B. The fact that it has been five years is almost unbelievable to Drew. This emphasizes the amount of time that has passed, and also shows us how Drew feels about this passage. It also lends gravity to the fact that during those years, a great deal has transpired. This doesn't tell us anything about the author.
8. H. The key word in this sentence is "luxuriate." This means to relax and experience pleasure and enjoyment. Therefore, we can infer that the surroundings are pleasantly warm and relaxing. This tells us that the setting is not "unbearable" or too harsh for plants to grow. There is also nothing in this sentence to indicate how much farther Drew has to travel.
9. D. The "bite of last winter's cold" feels permanent to Drew, and is a negative emotion. The fact that he doesn't think he'll ever warm up again is strongly negative, so we can rule out "gladness" and "pride." Though there may be regret about Boone's Fort, this paragraph deals primarily with Drew's difficult and challenging life experiences.

Adapted from *The Most Dangerous Game*

1. B. The speaker does not have a positive view of the alternatives offered to him after the "debacle in Russia." He says he is "lucky" to have alternatives so he would "never have to" open a tearoom.

2. E. In the first paragraph, the speaker states that he has hunted grizzlies, jaguars, and buffalos, but not lions.
3. C. In the passage, General Zaroff is the same person as “the Cossack.” This is the same speaker in the first paragraph, and is the person we learn that Rainsford addresses.
4. G. General Zaroff does not hunt for food, or for trophies. Instead, he is a hunter to chase the animals. He states that he enjoys “the problems of the chase.”
5. C. The speaker of the first paragraph is the Cossack, or General Zaroff. He detests perfection, since it is boring. This is confirmed when he says that it was “a tragic moment” because “there is no greater bore than perfection.”
6. F. A “sporting proposition” is something that is fair and challenging and conducted for the enjoyment and experience, not a “bore” or a “sure thing.” The other choices describe examples of a “sure thing” or something unfair.
7. A. For the general, the problem with most animals is that animal instincts are “no match for reason.” In this case, “reason” is used to mean intelligence, or sentience. Therefore, the general would want to hunt something that is not just fast, powerful, or dangerous, but intelligent.
8. F. Rainsford, listening to the general, is at first just a listener, occasionally chiming in to ask Rainsford a question. However, as the general reveals that he has invented a new animal to hunt, Rainsford becomes more and more intrigued. Rainsford insists that the general tell him more about the animal, and eventually gasps. The author uses this to suggest that Rainsford is shocked and no longer just a casual listener, but actively concerned, or worried.

Adapted from *The Cruise of the Dazzler*

1. A. Joe is described as being “demure” (shy) and “nervous” (since he was fiddling). This means that he was expecting his father to be something negative, like critical, furious, or disappointed. One would not be shy or nervous if one knew that someone was going to react patiently.
2. F. The passage tells us that “the gold” was recovered, implying that the contents of the safe included gold. Joe’s father gives him “credit, and plenty of it” to Joe for recovering the safe. We know that Joe was successful in his trip, and recovered the safe, but don’t know who the ‘Frisco Kid is.
3. C. Joe said that the outcome “couldn’t possibly have turned out any better,” implying that he thought he had done his best. However, his father’s reaction was deliberate and judicious, as well as reserved. This leads Joe to be disappointed, since he was hoping his father would agree and share in the triumph.
4. G. In some cases, “qualified” means “experienced” and “capable.” In this case, however, we can tell that the word is negative. This rules out “expert” and “competent.” “Absolute” doesn’t work, because we know that Joe felt disappointment, and therefore, he did not receive “absolute approval.” Instead, his father was reserved, and offered partial, or incomplete, approval.
5. B. The most likely choice is that the safe belong to Joe’s father, since he was the one spending money (in the form of an award) to recover it. We don’t know the amount of money in the safe.
6. G. Mr. Bronson, Joe’s father, launches into a big speech in the second half of the passage, telling Joe that life and experiences are invaluable; one would not trade one’s life for a million dollars. This is to say that even if an outcome is not as good as hoped for, that this doesn’t mean the experience was pointless.
7. D. In the aforementioned lines, Mr. Bronson speaks with a great deal of care. Otherwise, he would not go to such lengths to explain himself, ask rhetorical questions, or demonstrate his care about Joe’s safety and life experiences.
8. E. The phrase “potent for evil as for good” suggests that it is too early to tell whether Joe’s experience was good or bad. This is supported further when he says “until time has told me, whether...”

Excerpt from *The Innocents Abroad*

1. A. The phrase refers to the waves in the ocean and is explicitly discussing the rough seas that are keeping the boat from traveling.
2. H. “Repetitions” and the fact that the passengers were “just as eligibly situated as we could have been anywhere” means they had nowhere to go, and nothing to do. Thus, this describes the author’s boredom. The fact that he indicates that he is “just as eligibly situated as we could have been anywhere” suggests that he is patient, not impatient.
3. C. Paragraph 2 describes the narrator’s musings on breakfast, which is a time, he believes, when people can be vulnerable. During this time, people are more relaxed, if they ever are, and their true nature can be observed. This implies that normally, people are guarded, or masked, covering up their true nature.
4. H. The simile in question is “a spirit of charity rose up in their place that was as boundless...as the broad ocean”. Here, the comparison is made between the amount of charity and the vastness of the ocean. This represents a very positive, or optimistic, point of view. We can rule out “dread” and “nervousness,” which are negative feelings that have disappeared. The narrator is selfless, but not because he wants to help passengers (there’s no indication of that at all); remember that we must read paragraph 4 in its entirety to determine context.
5. D. The narrator is so happy he could sing, except he doesn’t. He may be certain about this fact, but it’s certainly not something he boasts of. Instead, the sentence should be read as tongue-in-cheek – a self-deprecating comment about how he is sparing his fellow passengers his singing.
6. H. The author is not selfless or particularly considerate of others. He greets them, but he cares more about the fact that their seasickness makes him feel better about himself. He acknowledges this fact, but this doesn’t make him any less self-involved.
7. C. The “good pleasure” the narrator feels is mentioned right after the experience of interacting with many miserable people. The contrast between how he feels (positively) and what he experiences (other people’s negative feelings) tells us more about the narrator’s character.
8. H. The sentence is an excellent example of imagery that describes the violence of the seas. The quote describes the front and back of the ship pitching and yawing in the rough waters. The phrase “taking a deadly aim at the sun in midheaven” suggests that the front of the boat is nearly vertical one moment, while the next it is “trying to harpoon a shark in the bottom of the ocean” suggests that it is nearly vertical the opposite way.
9. D. The central idea of the sentence is that a person will feel proud if he or she is not seasick when others around them are. There is a feeling of superiority that is inherent in such a belief. This is echoed in the last choice, which describes how the narrator is glad that others felt badly because he liked it. It made him feel better than others.
10. G. These paragraphs describe the narrator’s interaction with others. Time after time, he meets other people who say “Oh my” as they feel sick from the roiling waters. This makes the narrator gleeful, ultimately, that he is superior to others and does not succumb to seasickness.

Adapted from *The Luckiest Girl in the School*

1. B. The argument at the center of the passage is about where Winona will attend school, and Ms. Woodward is on the edge of capitulating to Winona’s demands. The excerpt shows that it took Mr. Joynson’s reassurances to convince Ms. Woodward to stick to her intention; hence, she is depending on others to make a decision. Although it is true that Ms. Woodward’s indecision prolongs the debate, this excerpt specifically is not addressing that.
2. F. As Percy goads Winona about how she will do terribly at school, he uses many mocking words, calling her a “pig,” and a “whipped puppy with its tail between its legs.” Though it is possible that he is doing this “to comfort” her (paragraph 10), his tone cannot be described as “encouraging.”

3. C. In this moment, Winona's little sister, Letty, is listing all the things she is looking forward to in Winona's absence, and she seems happy that Winona will be leaving. Percy's comments from paragraph 8 show no reluctance to let Winona leave, even if he does not think she will succeed while away.
4. H. The Aesop's story that she references, according to the footnote, is an example of a fulfilled request not turning out as positively as the requester expected. In this context, Winona is using it to say that her absence may not be a good thing, since the younger children will then have to obey a governess/teacher. She is expressing irritation at Letty's pleasure at her departure.
5. C. In paragraph 9, Winona's feelings about going away for school change from fully negative to mixed, since she now wants to prove to Percy that she won't be a failure. The argument between the siblings continued, and is not resolved. The excerpt ends before Winona goes away for school, so we cannot tell if her words were a correct prediction/foreshadowing.
6. F. Since she "set her teeth and clenched her fists," Winona's emotions cannot be characterized as "sorrowful;" instead, they show determination. Her reaction is to her siblings' comments, and not the adults in her life.
7. A. In paragraph 7, Winona is still complaining about having to go away. Paragraph 16 shows her resolved to succeed. Not enough time has passed for the comparison between the two moments to show how Winona has "matured" from a younger self.

## Numbers & Operations

### Numbers

1. B.  $\frac{1}{4} = 0.25$ .  $\frac{1}{3} = 0.33$ . Therefore, only  $0.33 > 0.31 > 0.25$ .
2. E. A repeating decimal never ends. We know that  $\frac{1}{3} + \frac{1}{6}$  is equal to  $0.\overline{33} + 0.1\overline{66}$ , which equals 0.5. If we were to add the fractions, we would arrive at  $\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = 0.5$ . If we add any of the other fractions together, we end up with a non-terminating fraction.
3. C. The only way for the product of one number and another number to equal one of those numbers is for one of the numbers to be equal to 1. We can prove this by using examples. Let  $x = 10$  and substitute into the equation:  $10m = 10$ . By dividing both sides by 10, we can see that  $m = 1$ .
4. F. We can express these inequalities as follows:  $p + 7 < 0 < p + 9$ . Try each answer choice to find that only  $-8$  makes a true statement.
5. B. If the three different numbers are multiplied together to get 15, the only way to do this is to multiply 5 by 3 by 1. This is because 15 only has the following factors: 1, 3, 5, and 15. If we have to use 3 different numbers, the only way to do so is to use 1, 3, and 5. The sum, then, is  $1 + 3 + 5 = 9$ .
6. G. We are looking for two square numbers that have a difference of 11. Going through the square numbers (1, 4, 9, 16, 25, 49, 64, etc.) we see that only 36 and 25 have a difference of 11, meaning  $x^2 = 36$  and  $y^2 = 25$ , so  $x = 6$  and  $y = 5$ .
7. A. We know that Alice's speed is twice Bob's speed, which is 9 mph. This means Alice is biking at 18 mph. If Carol's speed is one-third of Alice's, then her speed is 6 mph.
8. E. Since the question asks for the least amount the author could pay the editor, we can assume that the editor is reviewing as many words per minute as possible, at the lowest rate possible. The editor will take  $10,000 \text{ words} \div 40 \text{ words per minute} = 250 \text{ minutes}$  to review the manuscript at his fastest possible speed. This translates to  $250 \text{ minutes} \div 60 \text{ minutes} = 4 \text{ hours and } 10 \text{ minutes}$ . If the editor is



paid the lowest rate of \$18 per hour, then the editor will make  $\$18 \text{ per hour} \times 4 \text{ hours} = \$72$  plus  $\$18 \text{ per hour} \times 10 \text{ minutes}$  (or  $\frac{1}{6}$  of an hour) = \$3. So, a total of \$75.

9. C. Consulting the chart, we see that there are a total of 32 plants with white flowers, 19 of which already contain yellow seeds. There are 23 plants with yellow seeds. Thus,  $32 + 23 = 55$ .
10. G.  $z$  represents the total of all flowers in the experiment, which can be found by adding all mutually exclusive values. We can add  $x + y$ , or we can add  $n + r$ , or we can add  $k + m + p + q$ . They will all add up to  $z$ . As shown in the Plugins section, you can also solve this by replacing the variables with numbers and seeing which answer choice works with your chosen values.
11. A. We can start filling out the rest of the chart by filling in any row or column in which there is only one blank space. The intersection of Latin and Geometry is 12,000 ( $84,000 - 72,000$ ), and the total number of mandarin students is 17,500 ( $101,500 - 84,000$ ). We can already now see that, if the total number of Geometry students is 12,500 and 12,000 of them are taking Latin, then that mean only 500 Geometry students are taking Mandarin.
12. G. An integer is a whole number. The word “inclusive” means to include the numbers in the range itself. So, we have 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20: 11 integers. A rule for determining the number of items in an inclusive range is to take the difference of the upper and lower bounds of the range and add 1.
13. B. Convert the fractions provided in the question into standard form:  $7 \div 3 = 2.33$  ( $-2.33$  for the negative fraction). From this, we know that there are 5 whole numbers (integers) between (greater than)  $(-2.33)$  and (less than)  $2.33$ :  $-2, -1, 0, 1$ , and  $2$ . Note that both fractions result in non-terminating (repeating) decimals, but we don’t need to worry about those since we are concerned with integers.
14. G. When describing a range, the word “inclusive” means to include the end numbers in the range itself. The word “exclusive” means to exclude the end numbers in the range itself. Set A contains all whole numbers from 50 to 100 (including 50 and 100), and Set B contains all whole numbers from 69 to 138 (excluding 69 and 138). Another way of saying this is that Set B includes all whole numbers from 70 to 137, inclusive. We know that all numbers from 70 to 100 inclusive are in both Set A and B.  $100 - 70 + 1 = 31$ .
15. C. There are 49 positive integers under 50 (1 through 49). To find out how many are not multiples of 4 or 6, we must find out how many integers are multiples of 4 or 6. There are twelve multiples of 4 under 50 and 8 multiples of 6 under 50. That would make a total of twenty, but there are overlaps, namely 12, 24, 36, and 48. Making sure those numbers are not counted twice, that leaves us with sixteen multiples to take out. We are left with 33 integers.
16. E. A three-digit integer is any number from 100-999 inclusive. The question tells us to consider only numbers in the 700 range, from 700-799 inclusive. It also tells us to consider only integers where the units digit equals 1. This can only happen once in every set of 10: 701, 711, etc. Therefore, there are a total of 10 integers fitting the criteria described by the question.
17. D. If the values of the coins Dimitri has are 1, 5, and 10 cents, we need to list out different ways for the sum to equal 17. Using the dime, there are 2 ways to make 17 cents:  $10 + 5 + (2)1$ , or  $10 + (7)1$ . If we don’t use the dime, we can use just nickels and pennies:  $3(5) + 2(1)$  or  $2(5) + 7(1)$  or  $5 + 12(1)$ . If we don’t use the nickels or dimes, and just use pennies, we can use  $17(1)$ . Therefore, there are 6 different ways to solve the question.
18. H. We must work backwards from the current share price of \$25 up to the price of \$30. The price has been dropping at a rate of \$0.50 per half hour, or \$1 per hour. Since there is a difference of \$5 between the price now and the price before the price began dropping, and since we know that the price drops \$1 per hour, then we know that it has taken 5 hours for the price to have dropped \$5.

19. D. Since the numerator in the fraction is 1, we know that the larger the denominator, the smaller the value of the fraction. Since there is a finite range for  $y$  (between 1 and 5, inclusive), we know that the smallest value for the denominator will give us the largest value of the fraction. Thus,  $1 \div 1 = 1$  (as opposed to  $1 \div 5 = 0.2$ ).
20. G. Substitute the values of the answer choices and simplify.  $7(4) \div 4 = 7$ , and the square root of 7 is not an integer. Similarly,  $7(7) \div 4 = 12.25$ , which is not an integer, so its square root will not be an integer.  $28 \div 4 = 7 \times 7 = 49$ , which does simplify to an integer when you take the square root ( $7 \times 7 = 49$ ).
21. C. Remember to resolve terms raised to an exponent before multiplying terms. In this case, this means to substitute  $-1$  in for  $p$  and raising it to the power indicated in each answer choice before multiplying by the coefficient in front of  $p$ . A negative number that is raised to an even number will always result in a positive number (while a negative number that is raised to an odd number will always result in a negative number). This is because a negative number times a negative number (in this case, itself), will always be a positive number, but a positive number times a negative number will always be negative. Thus,  $8(1)$  is greater than all other answer choices.
22. E. Use substitution to try each possible value of  $c$ . If we substitute 1, we arrive at  $b = \left(-\frac{1}{2}\right)^1 = -\frac{1}{2}$ . If we square  $b$ , we arrive at  $\left(-\frac{1}{2}\right)^2 = \frac{1}{4} = a$ . If we substitute in 3, we arrive at  $b = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$ . If we square  $b$ , we arrive at  $\left(-\frac{1}{8}\right)^2 = \frac{1}{64} = a$ . Since  $\frac{1}{4} > \frac{1}{64}$ , then the greatest value for  $a$  must be when  $c = 1$ .  
Remember also that the more times we multiply a fraction by a fraction, the smaller the resulting value. Since the exponent means to multiply a fraction by itself, we know that if we keep doing this, the resulting value will be smaller and smaller.
23. B. When a negative number is squared (raised to the second power), the result is positive, making it larger than the original number. Since the number line shows that  $x$  is greater than  $x^2$ , the negative answer choice will not work. We know that  $x$  cannot equal 1, because then  $x = x^2 = x^3$ . We can test out both remaining fractions. If we square  $\frac{4}{3}$ , we will get  $\frac{16}{9}$ , which is greater than  $\frac{4}{3}$ . This means that  $x^2 > x$ , which we know cannot be the case given the number line. Therefore, only  $\frac{2}{3}$  works, as the square of  $\frac{2}{3}$  will be  $\frac{4}{9}$  making  $x > x^2$ .
24. E. Substitute the values of  $x$  into the different answer choices. Be sure to test at least two values of  $x$ . 0 is the simplest to test. We see, if we test 0, that both  $x^2 + 1$  and  $2x^2 + 1$  are valid, leaving us with a  $y$  value of 1. 1 is an easy value to test also. If we test 1, we see that only  $x^2 + 1$  is valid.
25. A. Test the relationship between  $Y$  and  $X$  at various points in the table. By doing so, we can see that  $Y$  represents the square of the  $X$  value. For example,  $2^2 = 4$ , and  $6^2 = 36$ . Therefore, if 100 appears in  $Y$ , the  $X$  value must be 10, since  $\sqrt{100} = 10$ .
26. F. Since  $y$  is a positive integer, we can automatically rule out  $x = 3$  or 4 because the product of each term times these numbers would exceed the value of the next term (for example,  $9 \times 3 = 27$ , which is greater than 21; this might work if  $y$  were allowed to be negative). We can test out 1 or 2. If we plug in 1 for  $x$ , then  $y$  would have to be 12 if the first term is 9 and the second term is 21 ( $21 = 9(1) + y$ ). However, if we do this for the second term, we see that the equation doesn't work:  $21(1) + 12 = 33$ , which is not 45. Trying  $x = 2$ , we see that  $9(2) + y = 21$ , and  $y = 3$ . For the second term,  $21(2) + 3 = 45$ , which works.

27. B. We must add together the sum of students who are taking Biology or Chemistry, as well as those who take neither Biology nor Chemistry. This is  $16 + 8 + 2 = 26$ . Then, we should subtract the number of students who take both Biology and Chemistry. This is because they are also counted in the number of students who take Biology or Chemistry. The total number of students is therefore  $26 - 4 = 22$ .
28. G. Substitute values in for each value of  $z$ . When we do this, we see that only when we plug in 13 for  $z$  is this true:  
 $2(13) + 7 = 33$ , and  $33 \div 5 = 6 \frac{3}{5}$ . The numerator tells us that the remainder is 3.
29. C. If there are 2 nickels for every 1 penny in the piggy bank, then  $\frac{1}{3}$  of all coins in the piggy bank are pennies, and  $\frac{2}{3}$  are nickels. Since this is the case, we know that the total number of coins must be evenly divisible by 3, otherwise, we would have a remainder/partial coin, which would be impossible. All choices are evenly divisible by 3 except for 55.
30. E. If Stanley is the 10<sup>th</sup> tallest and 10<sup>th</sup> shortest student in the class, then he represents the median height of his class. This is because there are 9 students who are taller than him, and 9 students who are shorter than him. This is a total of 18, plus Stanley himself, which makes 19.
31. C. If the program selected 51, then 51 would be printed, not 102. If 102 was selected, then 51 would be printed. Only selecting 204 would result in printing 102.
32. F. There can only be one number between 1 and 12 in the combination, as that represents the month of Tina's birthday. Therefore, we can rule out the other choices immediately. The last choice is incorrect because 11 is both the only odd number and the only number that could represent Tina's birthday month. 10 represents the month of Tina's birth, 15 the odd number, and 24 the factor of 24.
33. 210. There are 10 pairs equal to 21.  $1 + 20 = 21$ ,  $2 + 19 = 21$ , and so on, up to  $10 + 11 = 21$ .  $21(10) = 210$ .
34. -4. The important words here are "integers" (which means whole number) and "exclusive" (which means choose the next integer within the range). In this case, we should sum all integers between and including -4 and 3. Since positive and negative integers from 3 and 3 cancel, the only integer remaining is -4.
35. 14. Square all three parts of the inequality:  $49 < x < 64$ . All integers from 50 to 63 are acceptable values of  $x$ . There are 14 solutions.
36. 12. Either list them out as 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60, or do a prime factorization tree to see that  $60 = 2^2(3)(5)$ . Adding one to each exponent and multiplying these new numbers yields  $3(2)(2) = 12$ .
37. 120. If  $n$  is divisible by 3, 4, and 5, it must have 3, 4, and 5 as factors. The least common multiple of these numbers is 60. Since the question states that  $n$  must be greater than 100, the next common multiple greater than 60 is 120.
38. 19.  $1,600 = 40^2$ .  $3,600 = 60^2$ . In between, there are the squares of the positive integers from 41 through 59. There are 19 perfect squares in total.

## Operations

- D. Observe proper order of operations (PEMDAS). Multiply fractions first, and do so by multiplying across the numerators and denominators. This gets us to  $\frac{3}{8} + \frac{1}{6}$ . To add, we find equivalent fractions that have common denominators, so  $\frac{3}{8} + \frac{1}{6} = \frac{9}{24} + \frac{4}{24}$ . Now, we can simply add across the numerators since the denominators are the same:  $\frac{9}{24} + \frac{4}{24} = \frac{13}{24}$ .
- G. Following the correct order of operations, we must divide  $\frac{3}{4}$  by  $\frac{1}{6}$  first, which we can rewrite as  $\frac{3}{4} \times 6$ , which equals  $\frac{18}{4}$ , or  $\frac{9}{2}$ . Adding  $\frac{1}{2}$  to that results in  $\frac{10}{2} = 5$ .
- B. Notice that all answer choices use the same digits, and only the decimal place changes. Since  $2 \div 1 = 2$ , we know that the answer must be close to 2. Thus, 1.95.
- H. Notice that all answer choices use the same digits, and only the decimal place changes. We can use logical reasoning to know that  $73 \div 2 = 36.5$ . Since the decimal in 36.5 is two places from where it should be, we know we must move the decimal in 2 two places to the right, giving us 200.
- D. When dividing a number by a fraction, we can multiply that number by the fraction's denominator then divide by the fraction's numerator. In this case,  $2.4 \times 6 \div 1 = 14.4$ .
- F. Following order of operations, we divide 2.02 by 0.01 first. We can move both decimal points over two places, meaning we are now dividing 202 by 1, which is 202. Multiplying that by 0.03 gives us 6.06.
- C. Remember proper order of operations: PEMDAS. Resolve terms inside of parenthesis, then exponents, then multiplication and division before addition and subtraction.
- F. We know that  $\sqrt{49} = 7$  and  $\sqrt{64} = 8$ , so  $\sqrt{50}$  and  $\sqrt{60}$  are both between 7 and 8. Therefore, the sum must be greater than 14 and less than 16. The only reasonable range is between 14 and 15.
- C. The most efficient way to solve is to go through each answer choice, observing the correct order of operations. For choice A, solve the operation inside the radical first, giving you  $\sqrt{11^2}$ , which is equal to 11. For choice B, solving the operations inside the radical results in  $\sqrt{121}$ , which also equals 11. For choice C, the operations inside the radical result in  $\sqrt{25 + 36}$  which equals  $\sqrt{61}$ . We may not know the exact value of  $\sqrt{61}$ , but we know that  $\sqrt{49} = 7$  and  $\sqrt{64} = 8$  so  $\sqrt{61}$  must be between 7 and 8.
64. The sequence is made of cubes of integers.  $0^3 = 0$ ,  $2^3 = 8$ , etc. Since  $3^3 = 27$ , the next term is  $4^3 = 64$ .
2. Set the two equations for  $x$  to equal each other.  $14 - 4y = y - 1$ . This means, upon adding  $4y$  and 1 to both sides of the equation,  $15 = 5y$ , so  $y = 3$ . Substituting this value into either original equation yields  $x = 2$ .
- 0.05. Remembering PEMDAS, we know that multiplication and division go first, so  $1.2 \div .25 = 1.6$ , and  $2.5 \times 0.3 = 0.75$ , giving us  $0.8 - 1.6 + 0.75 = -0.05$ .
- 0.06. First, evaluate all multiplication and division operations. So, the equation becomes  $0.1 + 0.2 - 0.12 \div 0.5$ , and then  $0.1 + 0.2 - 0.24$ . Then, evaluate from left to right:  $0.3 - 0.24 = 0.06$ .
- 5.12. Observe PEMDAS. Simplify inside the parenthesis for  $3.2^2 - 15.36$ . This simplifies to  $10.24 - 15.36$ , or -5.12.
- 0.5. Observe PEMDAS. Converting to improper fractions may help.  $4.2 = 4\frac{1}{5}$ , or  $\frac{21}{5}$ . Multiplying by  $\frac{5}{6}$  gives us  $\frac{105}{30}$ . Dividing a fraction by an integer is the same as multiplying that fraction by the



integer's reciprocal, or  $\frac{105}{30} \times \frac{1}{10} = \frac{105}{300}$ . Simply multiply both the numerator and denominator of  $\frac{3}{20}$  by 15 to end up with  $\frac{45}{300}$ , which we can add to  $\frac{105}{300}$  and end up with  $\frac{150}{300} = 0.5$ .

### Percents

- A. The first thing to notice is that you do NOT have to actually find the numerical value of the expression. The answer choices tell you to simplify, not solve. It's easier to find  $\frac{1}{3}$  of 924 rather than 35% of 924. Simple division tells us that  $924 \div 3 = 308$ . The equivalent of the statement is 35% of 308.
- F. Green was the favorite color for 35% of the students and there were 3,000 students total, so we must find 35% of 3,000. This is  $0.35 \times 3,000 = 1,050$ .
- D. If there are 12 boys, then the number of girls is  $30 - 12 = 18$ . To figure out what percent this is, take the number of girls and divide by the total number of students:  $18 \div 30 = 0.6 = 60\%$ .
- G. If there are 200 pencils in a case, and 75 are sharpened, then 125 pencils are not sharpened.  $125 \div 200 = 62.5\%$
- B. In total, there are  $1.5x + x + 2x + 3x + 0.5x = 8x$  students in the school. Of this,  $x$  number of students own 1 video game. This is  $1 \div 8 = 0.125$ , or 12.5%.
- G. By interpreting the chart, we can see that people under the age of 35 consists of 20% + 25% of the building population. 45% of 1,000 = 450.
- C. The overlapping circles with a value of 8 tells us that 8 students studied both Spanish and French. Since the question asks the number of students studying French (not exclusively French), then this is  $5 + 8 = 13$ . Since there are 25 students in total,  $13 \div 25 = 0.52 = 52\%$ .
- H. We want to find the number of students who own 2 pets or more. 25% of the students own 2 pets, 8% own 3 pets, and 2% own 4 or more pets. In total, this makes 35%. We must calculate 35% of 2,400 which is  $0.35 \times 2,400 = 840$ .
- C. If 20% to 30% of the students go on to a master's program, then 70% to 80% will not go on to a master's program. Therefore, the minimum that will not go on to a master's program is 70%. Calculate 70% of 4,000 students, which is  $0.70 \times 4,000 = 2,800$ .
- F. It rained on 40% of the days which is  $0.40 \times 365 = 146$  days. The number of days when it did not rain was  $365 - 146 = 219$ , or  $0.6 \times 365$ . The question asks: on how many fewer days did it rain than not? We must subtract the days when it rained from the days when it did not rain:  $219 - 146 = 73$  days.
- B. We can probably intuitively determine that the number of votes that Gillian received was 550, and the number of votes that Clive received was 450. However, let's solve this algebraically. Let  $x$  equal the number of votes that Clive received. In terms of  $x$ , Gillian received  $x + 100$ . So,  $x + x + 100 = 1,000$ . Simplify so that  $2x = 900$ , and  $x = 450$ . Thus, Clive received 450 votes, and if Gillian received 100 more, then she received 550. The percentage is simply  $550 \div 1,000$ , or 55%. We can eliminate a zero from both numbers to see that the remaining fraction is  $55 \div 100$ , or 55%.
- G. Let  $x$  represent the number of bottles Betty recycled. In terms of  $x$ , Bruce recycled  $2x$  bottles. In total, the number of bottles recycled by Betty and Bruce was 600, so  $600 = x + 2x$ , or  $600 = 3x$ . Solving for  $x$ , we find that Betty recycled 200 bottles, and Bruce recycled  $200(2) = 400$  bottles. This represents  $4 \div 6 = 0.667$ , or 66.7%.
- D. The initial area is  $50 \times 40 = 2,000 \text{ ft}^2$ . A decrease of 40% to the length of 50 ft. results in a new length of  $50 \times 0.6 = 30$  ft. An increase of 20% to the width of 40 ft. results in a new width of  $40 \times$

- $1.2 = 48$  ft. The new area is  $30 \text{ ft.} \times 48 \text{ ft.} = 1,440 \text{ ft}^2$ . This represents a total decrease of  $2,000 - 1,440 = 560 \text{ ft}^2$ , or  $560 \div 2,000 = 28\%$ .
14. G. Let  $x$  equal the original price of the laptop. If it was first increased by 10%, this can be represented as  $1.1x$ . If it was then decreased by 20%, we can represent this as  $0.8(1.1x)$ . Simply multiplying tells us that the final price is  $0.88x$ , or, in other words, 88% of the original price.
  15. A. If Kevin watched 2 hours of TV on Saturday, and he watched 100% as many hours on Sunday, there is no increase or decrease. Thus, he also watched for 2 hours on Sunday.
  16. E. The charge for shipping a \$49 order would be 15% of \$49, which is equal to  $0.15 \times \$49 = \$7.35$ . The charge for shipping a \$50 order would be 12.5% of \$50, which is equal to  $0.125 \times \$50 = \$6.25$ . The difference in the shipping charges is  $\$7.35 - \$6.25 = \$1.10$ .
  17. D. We are asked to find the total number of smartphones sold. Since 700 smartphones were sold other than Brands A, B, C, and D, the 700 smartphones that were sold were of Other Brands. Since Other Brands represents 20% of the total, then  $700 \div 20\% = 700 \div 0.2 = 3,500$ . We can check this by calculating 20% of 3,500 total sales:  $3,500 \times 0.2 = 700$ .
  18. H. A 150% increase is the same as multiplying by 2.5. If the baseline was 8 miles, then John ran  $8 \times 2.5 = 20$  miles on Wednesday.
  19. A. Starting with the second piece of information, we know that Y is 20% of 100, so  $Y = 20$ . X is 80% of Y, so  $X = 0.80 \times 20 = 16$ .
  20. H. The half-life of fermium-253 is 3 days, meaning 50% of any amount of it decreases by 50% every 3 days. There are 3 such periods in 9 days. This means there is  $x(0.5)(0.5)(0.5)$  remaining after 9 days. This is a total decrease to 0.125, or 12.5% of the original amount. This means there was a  $100\% - 12.5\% = 87.5\%$  decrease. Use an easy number, like 100, to prove it out.
  21. D. When there are no concrete numbers given, use a number that is easy to work with. Let's say it is 100. At the end of one month, there will be  $100 \times 1.1 = \$110$  in the account. This is because 1 represents the entire value of the original amount (or 100%), and the 0.1 represents the 10% increase. After the second month, there will be  $110 \times 1.1 = \$121$ . This represents an increase of 21%, but the question asks for the percent of the original amount that is there, or 121%.
  22. E. 30% of the cost was due at signup, or  $0.30 \times \$1,200 = \$360$ . The amount that was left to pay after that was  $\$1,200 - 360 = 840$ . Then, 75% of this remaining balance was due the week before camp, or  $0.75 \times \$840 = \$630$ . Finally, the amount left to pay on the first day of camp was  $840 - 630 = 210$ .
  23. C. Of the first 600, if 30% are red, then  $600 \times 0.3 = 180$  are red. If the total number of red peppers must be 50% of the total, which is 1,000, then the red peppers must number at  $1,000 \times 0.5 = 500$ . Since there are already 180, then  $500 - 180 = 320$ , which is the number of peppers remaining that must be red.
  24. F. If the restaurant sold 50 steak entrées out of a total of 200, that means they sold 150 "other" entrées. If steak must represent a minimum of 50% of their total sales, they must have a minimum of 150 steaks entrées sold as well. If they've already sold 50, they need to sell another 100 to reach their goal.
  25. C. 80% of \$5.50 can be calculated as  $0.80 \times \$5.50 = \$4.40$ . The company charges 80% more than the original cost so we must add \$4.40 to \$5.50. The answer is \$9.90.
  26. G. Percent problems can be solved using proportions. Here, \$52 represents 130% of the amount the store paid to acquire the lamp, so we can set up the proportion  $\frac{130}{100} = \frac{52}{x}$ . Cross multiplying gives us  $130x = 5,200$ , so  $x = 40$ . If the employee discount is 40% off the this amount, then we can find 40% of \$40, which is \$16, and subtract that from the \$40 cost, which leaves \$24.
  27. 64. 20% of 100 is 20, so at first 80 chickens remain. 20% of 80 is 16, so finally  $80 - 16 = 64$  chickens are left.

28. 80. There are  $16 + 12 + 8$  wild dogs in the zoo, or 36 wild dogs. Set up a proportion to determine the total number of animals in the zoo. If  $\frac{36}{x} = \frac{45}{100}$ , where  $16 + 12 + 8 = 36$  total wolves, coyotes, and dingoes, and  $x$  represent the total number of animals in the zoo, then we cross-multiply to get  $3,600 = 45x$ , or  $x = 80$ .
29. 25. As a fraction in lowest terms,  $24\% = \frac{6}{25}$ . Since the number of students must be an integer, there could be any integral multiple of 25 students in the class. 25 itself is the smallest of these multiples, since multiplying 0.24 by any integer less than 25 will result in a decimal, or “partial” person (which doesn’t make sense).
30. 630. 30% of 1,000 is 300 ( $0.3 \times 1,000 = 300$ ) students who like the Scottish Fold. There are  $1,000 - 300 = 700$  remaining students. 10% of this would be  $0.1 \times 700 = 70$  students who like the Turkish Angora. This leaves  $700 - 70 = 630$  other students who have a different favorite breed of cat.

## Fractions

- D. A whole number can be rewritten as a fraction by putting the number in the numerator and a 1 in the denominator. So, we can express 6 as  $\frac{6}{1}$ . Similarly, we can express  $6\frac{2}{3}$  as  $\frac{20}{3}$ , by multiplying the integer 6 by the denominator 3 and adding it to the numerator 2. Now, we need to multiply across the numerator and denominator, and simplify:  $\frac{6}{1} \times \frac{20}{3} = \frac{120}{3} = 40$ .
- H. We can solve by converting both mixed numbers to improper fractions, giving us  $\frac{9}{2} \times \frac{18}{5} = \frac{162}{10}$ . We can use division to figure out that this equals 16.2
- D. A fraction can be represented as a decimal using division. Divide 44 by 64; we see that the fraction equals 0.6875.
- H. A fraction can be represented as a decimal using division. Divide 25 by 32; we see that the fraction equals 0.78125.
- B. We can solve this algebraically. We are told that  $\frac{2}{3}Q = 36$ . We can remove the denominator from the left side by multiplying both sides of the equation by 3. The equation then becomes  $2Q = 108$ . Dividing both sides by 2, we arrive at  $Q = 54$ .  $\frac{1}{3}$  of this is the same as  $54 \div 3 = 18$ . We can also understand that  $\frac{1}{3}$  is half of  $\frac{2}{3}$ .
- F. For two amounts to each represent half of the total amount, they must be equal. In this case, no additional oil is added, so there must be  $35 - 15 = 20$  tsp of vinegar added. This makes both amounts 35 tsp, which is 50% of the new total, 70 tsp.
- B. Let  $x$  represent the maximum tons of sand that can be contained in the cargo hold of a truck. In terms of  $x$ , the truck leaves the construction site at  $\frac{1}{4}x$  or  $\frac{x}{4}$  capacity. To this is added 9 tons of sand, making the total capacity  $\frac{5}{8}$  full. We can represent this as  $\frac{x}{4} + 9 = \frac{5x}{8}$ . We can combine like terms and solve for  $x$ . First, to subtract fractions, they must have the same denominator, so we multiply the fraction on the left by  $\frac{2}{2}$ :  $\frac{2x}{8} + 9 = \frac{5x}{8}$ . Then we arrive at  $9 = \frac{3x}{8}$  and remove the

denominator on the right by multiplying both sides by 8:  $72 = 3x$ . Dividing both sides by 3, we arrive at 24.

8. H. Since the tailor purchased each quantity of fabric in equal numbers, then we know that each lot of fabric cost him  $\$500 + \$275 + \$150 = \$925$ . We can divide his total purchase cost of  $\$5,550$  by  $\$925$  to determine that he purchased 6 lots in total. Since each lot contains 1 full square yard,  $\frac{2}{3}$  square yard, and  $\frac{1}{3}$  square yard, the total square yardage in each lot is  $1 + \frac{2}{3} + \frac{1}{3} = 2$ . 2 square yards per lot  $\times$  6 lots = 12 square yards.
9. B. Substitute the value of each subscript of X in for  $k$ . This becomes  $\left(\frac{2}{1}\right)\left(\frac{2}{2}\right) + \left(\frac{2}{2}\right)\left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)\left(\frac{2}{4}\right) - \left(\frac{2}{4}\right)\left(\frac{2}{5}\right)$ . We can multiply across each pair of fractions and simplify to  $\left(\frac{4}{2}\right) + \left(\frac{4}{6}\right) - \left(\frac{4}{12}\right) - \left(\frac{4}{20}\right)$ , or  $2 + \left(\frac{2}{3}\right) - \left(\frac{1}{3}\right) - \left(\frac{1}{5}\right)$ . Multiply each term such that the denominator equals 15:  $\frac{30}{15} + \frac{10}{15} - \frac{5}{15} - \frac{3}{15}$ . Simplify across the numerators:  $\frac{32}{15}$ .
10. H. If  $\frac{1}{6}$  of Pleasantburg taxpayers pay between \$10,000 and \$20,000 in taxes per year, then  $\frac{5}{6}$  do not.  $\frac{5}{6}$  of 2,850 =  $2,850 \times 5 \div 6 = 2,375$ . Alternatively,  $\frac{1}{6}$  of 2,850 =  $2,850 \times 1 \div 6 = 475$ , and  $2,850 - 475 = 2,375$ .
11. C. A sandwich 63 inches long is cut into thirds, which means each piece is now 21 inches long. One of these is cut into thirds again, which makes each of the resulting pieces 7 inches long. One of these is cut in half, making the resulting pieces 3.5 in. long. The question asks the difference between this and the longest pieces, which are 21 in. long. The difference is  $21 - 3.5 = 17.5$  in.
12. F. Jennifer's deposit of \$600 out of a total cost of \$2,000 is  $\$600 \div \$2,000 = 0.3$ , or  $\frac{3}{10}$ .
13. A. There are 24 hours in each day. In total, Tiffany drove 8 days  $\times$  24 hours per day = 192 hours. If she took a break for a total of 3 hours on each day for lunch (1 hour) and dinner (2 hours), then she took a break for a total of 8 days  $\times$  3 hours per day = 24 hours. We don't need to divide 24 by 192 to realize that the answer is  $\frac{1}{8}$ . This is because the 24 hours is equal to 1 day. So, out of the 8 total days, she took a break for 1 day. Alternatively, since 3 out of 24 hours is equal to  $\frac{1}{8}$  of the day, and Tiffany did the same thing each day, then  $\frac{1}{8}$  of the entire trip was spent on break.
14. F. Let  $x$  equal the denominator. In terms of  $x$ , the numerator can be expressed as  $x - 4$ . We could substitute answer choices for  $x$ , but this could be time consuming. Instead, we can solve this algebraically. If  $\frac{x-4}{x} = \frac{2}{3}$ , then we can cross-multiply, arriving at  $3x - 12 = 2x$ . Solving for  $x$ , we find that it equals 12. Thus, the denominator is 12.
15. 120. Using simplification by cancelling similar numerators and denominators, we get  $600\left(\frac{1}{5}\right)$ , or 600 divided by 5, which is 120.
16. 18. Multiply through by the common denominator 18 to obtain  $6x - 3x = 18 + 2x$ . Combining like terms, we get  $x = 18$ .



17. 5.5. The reciprocal of  $\frac{1}{5}$  is 5. The reciprocal of 2 is  $\frac{1}{2}$ . Added together, this gives us  $5\frac{1}{2}$ .
18. 0.05. Cancel out the entire numerator with the 6 in the denominator, and obtain  $\frac{1}{20}$ , which is also  $\frac{5}{100}$ , or 0.05.
19. 4. One-third is about 0.33, which is greater than 0.3.  $\frac{1}{4}$  is equal to 0.25, less than 0.3. Therefore,  $n$  must be at least 4.
20. 27. 4 people will take 3 times as long to do something as 12 people, so it would take  $12 \times 3 = 36$  days to wash 40 cars. Since 30 is only  $\frac{3}{4}$  of 40, it will take  $\frac{3}{4}$  of the time. So,  $\frac{3}{4}(36) = 27$ .

## Word Problems

- A. Profit can be defined as revenue minus expense. If Alan sells 30 clocks in one month at \$500 each, then he generates \$15,000 in revenue ( $300 \times \$500$ ). In one month, Alan spends \$2,500 on rent, which is an expense he would have to pay regardless of how many clocks he sells. However, it costs Alan \$275 to make each clock. For 30 clocks, it will cost Alan  $30 \times \$275 = \$8,250$  in total. Alan's total costs are  $\$8,250 + \$2,500 = \$10,750$ . We subtract this from his total revenue of \$15,000 and arrive at \$4,250.
- E. First calculate the cost per completed s'more. If each graham cracker costs \$0.50, and 2 graham crackers are needed, then  $2 \times \$0.50 = \$1$ . 1 marshmallow costs \$0.75, and 2 bars of chocolate cost \$2.50 ( $2 \times \$1.25$ ). So, one completed s'more costs  $\$1 + \$0.75 + \$2.50 = \$4.25$ . Since the counselor only has \$45, she can make 10 s'mores at a total cost of \$42.50 ( $10 \times \$4.25$ ). She can't make an 11th s'more because she only has \$2.50 remaining, after making the 10th s'more, which is not enough to buy the ingredients for the 11th s'more.
- C. The total change can be expressed as the difference between the two numbers. This means  $869 - (-299)$ . When we subtract by a negative, we add that number. So, we can rewrite this as  $869 + 299 = 1,168$ .
- H. This is a remainder problem. Since there are 271 guests, and each table can seat 8 guests, we can divide 271 by 8 to determine the number of tables needed.  $271 \div 8 = 33$  remainder 7. This means that there will be 33 full tables and 7 guests seated at the last table.
- C. The price of the pocketbook starts at \$800. The sale price can be calculated as  $\$800 \times 0.75$  (since the sale is 25% off). This means the sale price is \$600. If Jacquie has a coupon for another 30% off the sale price, we can express the final price as  $\$600 \times 0.70 = \$420$ . If sales tax is calculated on this price, then  $\$420 \times 0.09 = \$37.80$ .
- G. Since it is impossible to have a fractional part of a monkey, we know that the number of monkeys must be an integer. Therefore, we need an integer from which both 20% and  $\frac{1}{4}$  will result in integers. Simply going through the answer choices shows that only 40 works. With the other answer choices, finding either 20% or  $\frac{1}{4}$  results in an integer, but not both.
- C. Bao works 8.5 hours per day each day for 4 days. This translates to a total of  $8.5 \times 4 = 34$  hours. At \$25 per hour, he makes  $34 \times \$25 = \$850$  that week.
- F. The incremental cost to Ashley is \$0.02 per ounce. Since she drinks 11 ounces per glass, the incremental cost per glass is  $\$0.02 \times 11 = \$0.22$ . If she went to the gym 3 days that week, and she

drank 3 glasses of juice per day, then the incremental cost is  $3 \times 3 \times \$0.22 = \$1.98$ . There are still 4 days where she did not go to the gym. On those days, she only drinks 1 glass of juice. So the incremental cost on those days is  $4 \times 1 \times \$0.22 = \$0.88$ . For the week, the incremental cost to Ashley was  $\$1.98 + \$0.88 = \$2.86$ .

9. B. This question is simply asking about remainders. If we divide 546 by 36, we get 15 remainder 6. That means the new delivery person's 546<sup>th</sup> stop will be stop number 6.
10. F. If the depth of the water beneath the buoy decreases 6 inches each hour, it decreases by 1 foot over the course of 2 hours. Over 12 hours, this will happen 6 times ( $12 \text{ hours} \div 2 \text{ hours per foot} = 6 \text{ feet}$ ) for a total decrease of 6 feet. If the depth starts at 24 feet, and decreases by 6, the depth 12 hours later is 18 ft.
11. 16. We could try to find out how much each egg and strip of bacon costs, but we don't need to. If we add up the amounts in the first two sentences, we get 8 eggs and 5 strips of bacon cost \$8. The family ordered exactly twice as many eggs and twice as much bacon, so their cost will be  $8 \times 2 = 16$ .
12. 20. Dan can wash one car in one hour. His brother can wash two cars in one hour. Together, they can wash three cars in one hour. Therefore, they can wash one car in  $\frac{1}{3}$  of an hour, or 20 minutes.
13. 1,500. If Daphne earned \$80 in commission on a sale of \$1,600, then her percentage is  $\frac{80}{1,600} = 0.05$ , or 5%. Josephine earns twice the commission percentage as Daphne, or 10%. If her commission was \$150, then her sales must have been  $150 \div 0.1 = 1,500$ .
14. 100. By 5 p.m., Train A has been traveling for 5 hours at 80 miles per hour, or  $5 \times 80 = 400$  miles. Train B caught up to train A so it traveled the same distance, but it did so in 4 hours, so  $400 \div 4 = 100$  mph.
15. 12. If we let  $x$  = the number of quarters Frank has, then the number of dimes =  $37 - x$ . Since each quarter has a value of 0.25 and each dime has a value of 0.10, we can create the equation  $0.25x + 0.10(37 - x) = 5.50$ , which simplifies to  $0.25x + 3.7 - 0.10x = 5.50$ . Combining like terms gives us  $0.15x = 1.8$ . Solving for  $x$  results in  $x = 12$ .
16. 10.  $\frac{3}{8}$  of 32 is 12, and  $\frac{1}{2}$  of 32 is 16, so 12 students take French and 16 students take Latin. However, that is not a total of 28 students because there is overlap, as 6 students are taking both. Therefore, only  $28 - 6 = 22$  students are taking one or both of the languages, meaning  $32 - 22 = 10$  students are taking neither.

## Ratios & Proportions

1. A. Students must be familiar with the metric system.  $1 \text{ kg} = 1,000 \text{ g}$ , and  $1 \text{ g} = 1,000 \text{ mg}$ . Therefore,  $1 \text{ kg} = 1,000 \text{ g} \times 1,000 \text{ mg} = 1,000,000 \text{ mg}$ , and  $2,000,000 \text{ mg} = 2 \text{ kg}$ .
2. G. Any scale problem can be solved with proportions. Here, we can also set up a proportion:  

$$\frac{20 \text{ ft.}}{0.5 \text{ in.}} = \frac{60 \text{ yds.}}{x \text{ in.}}$$
However, the units must match, so we can convert yards to feet, giving us  $\frac{20 \text{ ft.}}{0.5 \text{ in.}} = \frac{180 \text{ ft.}}{x \text{ in.}}$ .  
Now we can cross multiply and solve for  $x$ :  $90 = 20x$ , so  $x = 4.5$ .
3. D. If salad dressing is 2 oz. vinegar and 5 oz. oil, then in total 1 serving of dressing uses 7 oz. liquid. The restaurant needs a total of 35 oz. of dressing, which is 5 servings of dressing ( $35 \text{ oz. needed} \div 7 \text{ oz. per serving}$ ). Since every 1 serving of dressing needs 5 oz. oil, we multiply 5 oz. oil per serving  $\times$  5 servings = 25 oz. oil.

4. E. We can represent the editor's proofreading rate in ratio form as 300 words : 9 minutes. The problem says he will proofread 1,000 words at the same rate. We can set up the proportion  $\frac{300 \text{ words}}{9 \text{ min.}} = \frac{1,000 \text{ words}}{x \text{ min.}}$ . Cross multiply and solve:  $9,000 = 300x$ , so  $x = 30$ .
5. B. We can set up a proportion to solve this problem just like all the others in this section, but we don't have to. Notice that all the answer choices use the same two digits – 1 and 9 – with the only difference being the placement of the decimal point. Looking at the given ratio, we see that items weigh roughly three times what they weigh on Mars. Knowing that, we can tell that that we are looking for a number that is roughly  $\frac{1}{3}$  that of 57. The only number that make sense is 19.
6. F. We can set up a proportion as follows:  $\frac{7 \text{ tigers}}{4 \text{ lions}} = \frac{28 \text{ tigers}}{x \text{ lions}}$ . Cross multiply and solve for  $x$ :  $112 = 7x$ , so  $x = 16$ . However, 16 is not the answer because the question asks for the total number of lions and tigers, not just the number of lions. So,  $16 + 28 = 44$ .
7. B. The question asks us to compare the average speed in miles per hour (mph) from two different trips. Miles per hour can be calculated by dividing the number of miles driven by the number of hours it took to drive that distance. On Monday,  $1,000 \div 25 = 40$  mph. On Tuesday,  $800 \div 16 = 50$  mph. The difference between Tuesday's and Monday's speeds is 10 mph.
8. G. There are several ways to solve this question, but the most straightforward way is to set up a proportion:  $\frac{40 \text{ newspapers}}{25 \text{ minutes}} = \frac{60 \text{ newspapers}}{x \text{ minutes}}$ . The first ratio is given, and the second represents what we are trying to determine. Note that the ratio does not use 100 newspapers as the numerator because the question asks us to determine the amount of time it will take to deliver the *remaining* newspapers. Cross multiply and solve for  $x$ :  $1,500 = 40x$ , so  $x = 37.5$  mins.
9. C. We need only to divide the total number of plogs and hucks by the amount that they convert to in terms of terps.  $200 \text{ plogs} \div 5 \text{ plogs} = 40 \text{ terps}$ .  $40 \text{ hucks} \div 0.2 \text{ hucks} = 200 \text{ terps}$ .  $200 + 40 = 240$ .
10. E. We can set up multiple proportions to solve this question. Because we know how many blue marbles there are, we can first use this and the second ratio to determine the number of yellow marbles:  $\frac{3 \text{ yellow}}{7 \text{ blue}} = \frac{x \text{ yellow}}{35 \text{ blue}}$ . Cross multiply and solve for  $x$ :  $7x = 105$ , so  $x = 15$ , the number of yellow marbles. Then use 15 in a second proportion:  $\frac{4 \text{ red}}{5 \text{ yellow}} = \frac{x \text{ red}}{15 \text{ yellow}}$ . Cross multiply and solve for  $x$ :  $5x = 60$ , so  $x = 12$ .
11. B. Though we could set up proportions to solve this question, we can recognize immediately that, in both ratios, the number representing the weight of almonds is the same: 7. We can think of the ratio as telling us about how the whole container is split into parts. There are, then, 5 parts peanuts, 7 parts almonds, and 3 parts walnuts, for a total of 15 parts. The total weight of the container is 75 oz., which is 5 times larger than 15; this means there are 5 times the amount of each type of nut than the one part. The container contains  $5 \times 5 = 25$  oz. peanuts,  $7 \times 5 = 35$  oz. almonds, and  $3 \times 5 = 15$  oz. walnuts. If we remove the weight of the peanuts, which is 25 oz., we get 75 oz. total – 25 oz. peanuts = 50 oz. remaining.
12. G. We are given two ratios and another value. We will set up our first proportion using the first ratio as it includes jelly beans:  $\frac{5}{9} = \frac{x}{12}$ , so  $9x = 60$  and  $x = \frac{20}{3}$  potato chips. Now we use that in our second proportion:  $\frac{2}{9} = \frac{20/3}{x}$ , so  $60 = 2x$  and  $x = 30$  carrots.

13. D. Remember that a ratio tells us how many parts there are of some total. In this case, there are 4 “parts” fiction books and 1 “part” nonfiction book, meaning there are 5 “parts” of books in total. Even if the ratio is represented in a fraction, for example  $\frac{4 \text{ fiction}}{1 \text{ non-fiction}}$ , the denominator does not represent the “whole,” as it does when working with non-ratio fractions. In this case, let’s just assume there are 5 books in total. This means there are 4 fiction books, which means 4 out of the 5 books are fiction.  $4 \div 5 = 0.8$ , or 80%.
14. G. We can solve using a proportion:  $\frac{8}{20} = \frac{x}{50}$ . Note that we use 50 instead of 30 because the problem says an additional 30 pounds is used. Cross multiplying, we get  $400 = 20x$ , so  $x = 20$ .
15. C. We can think of the diagram as two separate triangles, a smaller one with sides 6 and 2.4, and a larger one with sides 9 (because  $6 + 3 = 9$ ) and  $x$ . Since they share an angle and are both right triangles, they are similar triangles and therefore their sides are proportional:  $\frac{6}{9} = \frac{2.4}{x}$ . Cross multiplying results in  $6x = 21.6$ , so  $x = 3.6$ .
16. F. A ratio can normally be expressed as  $x \div y$ , in this case, girls  $\div$  boys. Since the number of girls is greater than the number of boys, this ratio will be greater than 1. We can use estimation to narrow down our choices. At Washington, Lincoln, Roosevelt, and Martin Luther King, Jr., the ratio of girls to boys is less than 3:1. We can tell this simply by multiplying the number of boys at each school by 3 to find that the number of girls there should be is greater than the number of girls that there actually are. For example, at Washington, a ratio of 3:1 girls to boys would mean that there should be  $1,500 \text{ boys} \times 3 = 4,500$  girls. In fact, however, there are only 4,000 girls. Only at Kennedy is the ratio more than 3:1 because  $1,020 \times 3 = 3,060$ ; since the actual number of girls at Kennedy is greater than this, Kennedy has the greatest ratio of girls to boys.
17. A. We can cross multiply, but we can also just notice that the numerator goes from  $m$  to  $2m$ , meaning the second ratio is double the first, so the value of  $z = 2(8) = 16$ .
18. G. The words “at the same rate” describe the use of proportions. Note that two different units are given for time. We must use matching units, so we’ll first convert 2 hours to 120 minutes:  $\frac{12}{120} = \frac{x}{40}$ . Therefore,  $120x = 480$  and  $x = 4$ .
19. D. Using cross multiplication, we get  $3x = 20$ , so  $x = \frac{20}{3}$ , which equals  $6\frac{2}{3}$ .
20. F. Any problem regarding the shadows of two objects in the same vicinity at the same time is referring to proportions. Here we can set up  $\frac{6}{20} = \frac{x}{120}$ . We cross multiply to get  $20x = 720$ , so  $x = 36$ .
21. B. Any problem describing similar polygons can be solved using proportions. We must make sure to set up the proportion using corresponding sides. Polygons will always be named using corresponding vertices. Here, we get  $\frac{8}{4} = \frac{6}{x}$ . Therefore,  $8x = 24$  and  $x = 3$ .
22. E. This is more of a logic problem. We are told that 2 hoses can fill a bucket in 5 hours. Therefore, 5 hoses should be able to fill the bucket in less than 5 hours. 2 is the only answer choice that is less than 5.
23. D. The words “for every” imply a rate is being given, so a proportion can be used to solve:  $\frac{16}{3} = \frac{x}{48}$ . We can cross multiply, but since 3 fits evenly into 48 exactly 16 times, we can simply multiply 16 by 16 to get 256.



24. F. We can set up a proportion, but we can also just notice that side  $FE$  is  $x$  and side  $BC$  is  $\frac{3}{2}x$ , meaning the second triangle is 1.5 times the size of the first, so the length of side  $AB = 1.5(8) = 12$ .
25. B. We are looking for the distance, so in order to set up our proportion, we need to know the elapsed time. There are 12 hours from noon to midnight, so  $\frac{500}{2} = \frac{x}{12}$ , which we cross multiply to get  $2x = 6,000$ , or  $x = 3,000$ .
26. E. This is more of a logic problem. We are told that 4 cows can eat all the grass in 10 days. If they add 1 more cow, then they should be able to eat all the grass in less than 10 days. 8 is the only answer choice that is less than 10.
27. C. There is not a proportional relationship described because a proportional value must include the ordered pair  $(0,0)$ . Here, when  $x$  is zero,  $y$  is 1,500  $(0, 1,500)$ .
28. G. The constant of proportionality is correctly shown being multiplied by the number of students,  $n$ , to determine the number of packs,  $p$ .
29. C. Since the values are proportional, we can determine that the constant of proportionality is 7. To find  $y$  when  $x$  is 5, we have to multiply 5 by 7 to get 35.
30. G. For a relationship to be proportional, it must pass through the ordered pair  $(0, 0)$ . Choice G is the only relationship that contains this point.
31. B. A proportional relationship will contain the point  $(0, 0)$ , although this point does not have to be shown in the table. For answer choice B, each  $x$  value is multiplied by 1.5 to obtain the  $y$  value.
32. 2,900. 0.2 fits into 500 2,500 times  $(500 \div 0.2 \text{ or } 500 \times 5)$ , so 500 drins = 2,500 rints. 0.5 fits into 200 400 times  $(200 \div 0.5 \text{ or } 200 \times 2)$ , so 200 vungs = 400 rints, for a total of  $2,500 + 400 = 2,900$  rints.
33. 30. If there are 48 tulips, then the 5:6 ratio of daisies to tulips can be multiplied to make 40:48, meaning there are 40 daisies. This means that the 3:4 ratio of roses to daisies must be multiplied by ten, resulting in 30 roses.
34. 25. First, determine amount of time it takes for Matthew to solve each question. We are told that he completed 60 questions in 75 minutes, or spent  $75 \div 60 = 1.25$  minutes per question. If there are  $80 - 60 = 20$  questions to go, he will spend an additional  $20 \times 1.25 = 25$  minutes finishing the test.
35. 1.2. The image shows two similar triangles, so their sides are proportional. The ratio of AE to AC is equivalent to the ratio of AD to AB, so  $\frac{8}{10} = \frac{4.8}{AB}$ . Cross multiplication gives us  $48 = 8(AB)$ , so  $AB = 6$ . However, the question asks for DB, which is  $6 - 4.8 = 1.2$ .
36. 7. The constant of proportionality is the value needs to be multiplied by  $x$  to get  $y$ . In this example each  $x$  value is multiplied by 7 to get the  $y$  value.
37. 1.2. Since a proportional relationship needs to be in the form of  $y = kx$ , and we are interested in the relationship between hours and miles we need to know what number can be multiplied by 2.5 to get 3. To find this, we must divide 3 by 2.5 to get 1.2 as the constant of proportionality.
38. \$125. Since there are two months that Martin would have deposited money, we can take \$250 and divide it by 2 to figure out how much he deposited each month. The result is \$125.

## Factors, Multiples, Exponents, & Radicals

1. C. A multiple is the result when two numbers are multiplied together; a common multiple is one that is shared between different numbers. In this case, the multiples of 450 are 450  $(450 \times 1)$ , 900  $(450 \times 2)$ , 1,350  $(450 \times 3)$ , etc. The multiples of 675 are 675  $(675 \times 1)$ , 1,350  $(675 \times 2)$ , 2,025  $(675 \times 3)$ , etc. We see that 1,350 is the first number that appears in both lists, so it is the least common

multiple. 5, 225, and 1,125 are not multiples of these numbers. 303,750 is a common multiple (which can be found by multiplying  $450 \times 675$ ), but it is not the least common multiple.

2. H. The multiples of 135 are 135, 270, 405, 540, etc. The multiples of 180 are 180, 360, 540, etc. We see that 540 is the first number that appears in both lists, so it is the least common multiple.
3. B. A common factor is any number that divides two numbers evenly. The largest of these is called the greatest common factor. In this case, it is 225. 5 and 25 are common factors, but they are not the greatest. 450 is not a factor of 675. 1,350 is a common multiple, not a common factor.
4. F. The only prime factors of 48 are 2 and 3, so 3 is the greatest prime factor. If we look at the other answer choices, 7 is prime but not a factor of 48 and 24 is a factor of 48, but is not prime.
5. D. The integers between 30 and 40 are 31, 32, 33, 34, 35, 36, 37, 38, and 39. An additional criteria tells us that the integers must be the product of two distinct (different) prime factors. Test each integer. 31 and 37 do not fit this criteria because they are already prime and cannot be the product of two different prime factors (1 is not a prime number). 33, 34, 35, 38, and 39 fit this criteria because  $3 \times 11 = 33$ ,  $2 \times 17 = 34$ ,  $5 \times 7 = 35$ ,  $2 \times 19 = 38$ , and  $3 \times 13 = 39$ . 32 and 36 have several factor pairs, but none of them are comprised of two distinct prime numbers.
6. F. The factors of 39 are 1, 3, 13, and 39. Of these, only 3 and 13 are prime (1 is not prime, and 39 itself is divisible by numbers other than 1 and itself, so it is also not prime). The factors of 20 are 1, 2, 4, 5, 10, and 20. Of these, only 2 and 5 are prime. Thus,  $x = 13$ , and  $y = 5$ , and the sum is 18.
7. C. The number of times that each factor occurs in the least common multiple is equal to the greatest number of times that it occurs in either of the original numbers. The least common multiple must have two 2's (because B has two 2's), it must have two 3's (because both A and B have two 3's), and it must have one 5 (because A has one 5). Another way to solve this problem is to multiply out the numbers:  $A = 90$ ,  $B = 36$ . The least common multiple of these numbers is 180.
8. E. The first bell will ring at 1:00, 2:00, 3:00, 4:00, 5:00, 6:00, 7:00, 8:00, 9:00, 10:00, 11:00, 12:00, 1:00. The second bell will ring at 1:30, 3:00, 4:30, 6:00, 7:30, 9:00, 10:30, 12:00, 1:30. The times when they both ring are underlined. There are 4 times in total when they both ring.
9. B. Express 27 in prime factors.  $27 = 3 \times 9$ , and  $9 = 3 \times 3$ . Therefore,  $27 = 3 \times 3 \times 3$ . Since we multiply 3 by itself 3 times, this is equal to  $3^3$ . Therefore,  $x = 3$ .
10. F. In an exponential expression like  $2^x$ , the base (which is 2) is multiplied by itself a certain number of times. The number of times the base is multiplied by itself is determined by the number in the exponent (in this case,  $x$ ). So in this case, 2 is multiplied by itself a certain number of times until the value is 32.  $2 \times 2 = 4$ ,  $2 \times 2 \times 2 = 8$ , etc. We can repeat this until we find that  $32 = 2 \times 2 \times 2 \times 2 \times 2$ . Since 2 is multiplied by itself 5 times over, the exponent is 5, or  $2^5$ .
11. B. Break down 80 into its prime factors. We know that  $80 = 8 \times 10$ . We can break this down further because  $8 = 2 \times 2 \times 2$  and  $10 = 2 \times 5$ . Therefore, we get  $80 = 2 \times 2 \times 2 \times 2 \times 5$ . Now write this in exponent form.  $80 = 2^4 \times 5^1$ . This shows that  $x = 4$  and  $y = 1$ . Therefore,  $x - y = 3$ .
12. E. Break down 225 into its prime factors. Since it ends in 5, we know it is divisible by 5. We get  $225 = 5 \times 45$ . Then we can break down 45 as  $5 \times 9 = 5 \times 3 \times 3$ . This gives the prime factorization  $225 = 5 \times 5 \times 3 \times 3$ . This can be expressed in exponential form as  $3^2 \times 5^2$ . Therefore,  $x = 2$  and  $y = 2$  so we get  $xy = 4$ .
13. A. Break down 900 into its prime factors. We get  $900 = 9 \times 100$ . This breaks down further to  $3 \times 3 \times 10 \times 10$ . The 10s can be broken down further, resulting in  $3 \times 3 \times 2 \times 5 \times 2 \times 5$ . In exponential form, this becomes  $2^2 \times 3^2 \times 5^2$ . Therefore  $x = 2$  and  $y = 2$  and  $z = 2$ , so we get  $x + y + z = 6$ .
14. E. Using prime factorization, we know that  $125 = 5 \times 5 \times 5$ , or  $5^3$ . If the bases of the exponential terms are already 5, then we know that term must be  $5^1$  and the other must be  $5^2$ , since if we multiply exponential terms, we add the exponents together. Therefore,  $x$  must be 1 or 2 (it doesn't matter which), and  $y$  must be the other. Therefore,  $(1)(2) = 2$ .

15. B. Using prime factorization, we know that 64 can be expressed as  $2 \times 2 \times 2 \times 2 \times 2 \times 2$  or  $2^6$ . In  $2^a \times 2^b \times 2^c$ , since we multiply each term, which has the same base, we are adding together the exponents. If  $64 = 2^6$ , then  $2^{a+b+c} = 2^6$ . But since each exponent must be different, they can't all be equal to 2. Instead, they are equal to 1, 2, and 3 (since  $1 + 2 + 3 = 6$ ). Thus,  $1 \times 2 \times 3 = 6$ .
16. F. The question asks which expression is equal to  $25^4$ . We know from prime factorization that 25 can be expressed as  $5^2$ , or  $5 \times 5$ . This means  $25^4$  is equivalent to  $(5^2)^4$ . When we raise an exponential expression (like  $5^2$ ) to another power (in this case, 4), we multiply the exponents. In this case, we end up at  $5^8$ .
17. C. Solve for  $a$  first. Since the bases are the same (all  $x$ ), we know that we must add the exponents together. Therefore,  $a + 6 = 24$ , and  $a = 18$ . Solve for  $b$  next. Since we are raising one exponent to another, and the bases are the same, we know that we must multiply the exponents together. Therefore,  $4b = 20$ , and  $b = 5$ . The sum becomes  $18 + 5 = 23$ .
18. F. We could simplify the expression by multiplying out the individual exponential expressions, but there is an easier way. When the bases of an exponential expression are the same, and we divide the exponential expressions, we simply take the difference in the exponent. Therefore,  $\frac{5^3}{5^2} = 5^1$  and  $\frac{2^5}{2^3} = 2^2$ . This becomes  $5(4) = 20$ .
19. B. There are several ways to solve this question. We could rewrite the expression so that the bases are the same, but it is perhaps simpler to write the expression in expanded form since the exponents share the same value (both 100 and 50 are raised to the 5<sup>th</sup> power).  $\frac{100}{50} \times \frac{100}{50} \times \frac{100}{50} \times \frac{100}{50} \times \frac{100}{50}$  is equal to  $2 \times 2 \times 2 \times 2 \times 2$ , or  $2^5$ , or 32.
20. F. There are several ways to solve this question, but again it is perhaps more simple to write the expression in expanded form using prime factorization. Notice that 7 and 2 are already prime, but that 28 can be expressed as  $7 \times 2 \times 2$ . This means that the expression given can be rewritten as  $\frac{7 \times 7 \times 7 \times 7 \times 2 \times 2}{7 \times 2 \times 2 \times 7 \times 2 \times 2 \times 7 \times 2 \times 2}$ . We can cancel out values in the numerator and denominator until we are left with  $\frac{7}{2 \times 2 \times 2 \times 2} = \frac{7}{16}$ .
21. A. Observe the proper order of operations. First, simplify under the radical, so that the expression is  $\sqrt{64}$ . The number that, when multiplied by itself gives you 64, is 8.
22. G. First simplify the fraction under the radical.  $100 \div 25 = 4$ . Then we take the square root of 4 which equals 2 because  $2 \times 2 = 4$ .
23. D.  $\sqrt{\frac{144}{81}} = \frac{\sqrt{144}}{\sqrt{81}}$ .  $\sqrt{144} = 12$  and  $\sqrt{81} = 9$ . Therefore,  $\frac{12}{9} = \frac{4}{3}$ .
24. E. In this case, if we square  $x + 3$  (multiply the term by itself), on the right side of the equation, we must also square the left side of the equation. Doing so would cancel out the radical (square root) sign and leave us with 5.
25. D. Solve for  $x$  first by subtracting 7 from both sides then squaring both sides to rid the left side of the radical. This becomes  $\sqrt{x} = 9$  and then  $x = 81$ .
26. H. The negative exponent means that each term, the numerator and the denominator, can be written on the other side of the divisor with a positive exponent. Then, raise each value to the positive form of the exponent.

27. C. The correct answer is I and II. Choice I results in a positive number because a positive integer raised to a negative exponent gives the reciprocal of the base raised to the positive exponent. Choice II results in a positive number because a negative number squared is positive. Choice III does not necessarily result in a positive number because if  $a$  is an odd number the result would be negative.
28. 2. In prime factorization, 64 can be represented as  $2 \times 2 \times 2 \times 2 \times 2 \times 2$ . It has no other prime factors except 2.
29. 121. If  $\sqrt{x-40} = 9$ , then  $x - 40 = 81$ , so  $x = 121$ .
30. 120. If Edina's tests have 30 questions, her total number of test questions could be 30, 60, 90, 120, 150, 180, etc. Patsy's total number of questions could be 24, 48, 72, 96, 120, 144, 168, etc. The smallest number they could have each had (i.e. the least common multiple) is 120.
31. 6. The prime factorization of 1,400 is  $2 \times 2 \times 2 \times 5 \times 5 \times 7$ , which can be written as  $2^3 \times 5^2 \times 7^1$ , so  $x$  is 3,  $y$  is 2, and  $z$  is 1. Therefore,  $x + y + z = 3 + 2 + 1 = 6$ .
32. -3. The prime factorization of 500 is  $2 \times 2 \times 5 \times 5 \times 5$ , which can be written as  $2^2 \times 5^3$ , so  $x$  is 2 and  $y$  is 5. Therefore,  $x - y$  can be expressed as  $2 - 5$ , which equals -3.

## Absolute Value

1. C. In the first absolute value,  $(-5) - 1 + (-3) = -9$ , so the absolute value is positive 9. In the second absolute value,  $2 - 7 = -5$ , so the absolute value is positive 5. When we add these together, we get  $9 + 5 = 14$ .
2. G. If we plug in the values  $x = 3$  and  $y = 9$ , we get  $5|3 - 9|$ . For the numbers inside the absolute value, we get  $3 - 9 = -6$ , so the absolute value is positive 6. Finally, we multiply this by 5 and get  $5 \times 6 = 30$ .
3. A. If we plug in the values  $x = -5$  and  $y = -15$ , we get  $|(-5) - (-15)| - |(-5) + (-15)|$ . Inside the first absolute value, we get  $(-5) - (-15) = -5 + 15 = 10$ . This is already positive, so the absolute value is still 10. In the second absolute value we get  $(-5) + (-15) = -20$ , so the absolute value is positive 20. Finally, we do  $10 - 20$  and get -10. This subtraction is not inside of an absolute value, so the answer stays negative.
4. G. When we take the absolute value of each number, they all become positive and we get  $9 + 9 - 9 \times \frac{1}{9}$ . Following the order of operations, we do multiplication before addition and subtraction.  $9 \times (\frac{1}{9}) = 1$ , so the expression becomes  $9 + 9 - 1 = 17$ .
5. B. Inside the first absolute value we get  $240 - 420 = -180$ , so the absolute value is positive 180. Inside the second absolute value we get  $24 - 42 = -18$ , so the absolute value is positive 18. This means that the equation becomes  $180 - 18 + a = 200$ . When we subtract the numbers on the left side we get  $162 + a = 200$ . Finally, subtract 162 from both sides of the equation. The final answer is  $a = 38$ .
6. F. Observe order of operations and simplify the equation to  $|-45| + p^2 - |-36| = 90$ . This becomes  $45 + p^2 - 36 = 90$ , and eventually  $p^2 = 81$ . Take the square root of both sides and  $p = 9$ .
7. A.  $|-3|$  is positive 3.  $|6|$  is positive 6.  $|-9|$  is positive 9. Therefore, the equation can be rewritten as  $K = -(-3 - 6 - 9)$ . For the expression inside the parentheses, we get -18. However, there is another negative outside the parentheses, so it becomes positive:  $K = 18$ . Finally, the question asks for  $-|K|$ . This becomes  $-|18| = -18$ . The answer stays negative because there is a negative sign outside of the absolute value.
8. H. There are two possible values for  $x$  that make the statement true. Either  $x = -5$  or  $x = 15$ . We are told that  $x$  is greater than 0, so  $x$  must be 15. However, the question asks for  $2x$ , which is 30.



9. A. If  $|x - 5| = 7$ , then  $x - 5 = 7$  or  $x - 5 = -7$ . Solving both equations results in  $x = 12$  or  $-2$ . If  $|y + 9| = 21$ , then  $y + 9 = 21$  or  $y + 9 = -21$ . Solving both equations results in  $y = 12$  or  $-30$ . Since  $x$  and  $y$  must both be negative,  $x = -2$  and  $y = -30$ . Therefore,  $x + y = -2 + (-30) = -2 - 30 = -32$ .
10.  $-40$ . Begin by plugging in  $-10$  and  $-20$ , giving us  $-|(-10) - (-20)| - |(-20) + (-10)|$ . Simplifying the expressions inside the set of absolute value bars gives us  $-|10| - |-30|$ . At this point, we can solve the absolute values, giving us  $-10 - 30$ , which results in  $-40$ .
11.  $-9$ . Simplify for  $J$ , giving us  $-5 - 5 + 5 \times \frac{1}{5}$ , which equals  $-9$ . Therefore,  $-|-J| = -|(-9)|$ , which equals  $-9$ .
12.  $-28$ . Remember that absolute value equations have two solutions:  $m - 8$  can equal  $12$  or  $-12$ , in which case  $m = 20$  or  $-4$ .  $n + 4$  can equal  $11$  or  $-11$ , in which case  $n = 7$  or  $-15$ . The problem states  $m < 0$  and  $n > 0$ ; we use  $-4$  for  $m$  and  $7$  for  $n$ , so  $mn = (-4)(7) = -28$ .
13.  $-9$ .  $|v - 7| < 17$  can also be written as  $-17 < v - 7 < 17$ , which simplifies to  $-10 < v < 24$ . Since  $v$  must be greater than  $-10$ , the smallest integer value that satisfies the statement is  $-9$ .

## Scientific Notation

1. A. Scientific notation expresses very large (or small) numbers by multiplying a number between 1 and 10 by 10 raised to some power. The exponent of 10 indicates the number of times the decimal point must be moved in order to produce a number in standard form. The sign of the exponent tells us which way to move the decimal point: positive means move the decimal to the right (increasing the number), and negative means move the decimal to the left (decreasing the number). In this case, 51,000 is written in standard form. In order to write this in scientific notation, we need to multiply a number between 1 and 10 by 10 raised to some power. In this case, we can use 5.1 as the number between 1 and 10, and multiply it by  $10^4$ . This is because from 5.1, we can move the decimal place “over” to the right 4 times, each time inserting a 0, since 10 raised to some power means that we’re multiplying by 10 over and over again.  $5.1 \times 10^4 = 51,000$ .
2. F. A number expressed in scientific notation is a number between 1 and 10 multiplied by 10 raised to a certain power (given by the exponent). In this case, the only number between 1 and 10 we can make from “0.513” is 5.13. From 5.13, we need to move the decimal place “over” to the left one place in order to make 0.513. Therefore, the exponent must be  $10^{-1}$ .
3. A. A number expressed in scientific notation is a number between 1 and 10 multiplied by 10 raised to a certain power (given by the exponent). In this case, the only number between 1 and 10 we can make from “0.0002088” is 2.088. From 2.088, we need to move the decimal place “over” to the left four places in order to make 0.0002088. Therefore, the exponent must be  $10^{-4}$ .
4. G. We know that the number between 1 and 10 should be 5.137. All we need to do is determine the number of times that we need to move the decimal place “over” to the right. Since  $10 \times 10 = 100 = 10^2$ , and  $10 \times 10 \times 10 = 1,000 = 10^3$ , etc., the number of times the decimal place needs to be moved depends on what number we need to multiply 5.137 by to get the number in standard form. To get 513,700, we would need to move the decimal point over 5 times, or multiply by  $10^5$ . However, the expression tells us to multiply 513,700 by 1,000, which is adding another 3 zeros. Thus, we should multiply 5.137 by  $10^{5+3}$ , or  $10^8$ . We can also see this is true as follows:  $513,700 \times 1,000 = 513,700,000$ . Moving the decimal point 8 places to the left results in 5.137.
5. C. If we multiply  $952.65 \times 1,000$ , we get 952,650. In scientific notation, this would be  $9.5265 \times 10^5$ , since we would have to move the decimal point 5 places to the right from 9.5265 to arrive at 952,650.

6. G. 10 raised to a negative power, like  $10^{-3}$ , lets us know that we need to move the decimal point “over” to the left a certain number of times. In this case, we move the decimal point over to the left 3 times. Since there is an implied decimal at the end of 5 (which can be written as 5.0), move the decimal place over 3 places, each time inserting a 0, arriving at 0.005.
7. B. 10 raised to a negative power, like  $10^{-7}$ , lets us know that we need to move the decimal point “over” to the left a certain number of times. In this case, we move the decimal point over to the left 7 times.
8. H. The distance between the two cities in km is  $4 \times 10^3$  which is equivalent to 4,000 km. There are 1,000 meters in 1 km, so the distance in meters is  $4,000 \times 1,000$  or 4,000,000 meters.
9. A. If the length of the building is 370 meters, it is represented on the blueprint by 370 mm. The question asks for the length in meters. To convert millimeters to meters, divide by 1,000, so  $370 \div 1,000 = 0.37$ , which is written in scientific notation as  $3.7 \times 10^{-1}$ .
10. F. If we needed to, we could write out both distances as 240,000 miles to the moon and 60,000,000 miles to Mars. We could then divide the distance to Mars by the distance to the moon and arrive at 250. An easier way is to divide both of the coefficients (the 2.4 and the 6.0) as well as the base and exponents (the 10 and their exponents).  $6.0 \div 2.4 = 2.5$ .  $10^7 \div 10^5 = 10^2$ ; when dividing exponents, simply subtract the numbers in the exponents when the bases are the same. We end up with  $2.5 \times 10^2$ , or 250.
11. C. One way to solve arithmetic with scientific notation is to rewrite the numbers in standard form.  $5 \times 10^3 = 5,000$  and  $3 \times 10^5 = 300,000$ , so their product is 1,500,000,000, which is equivalent to  $1.5 \times 10^9$ . If the problem involves multiplying or dividing of scientific notation, as this example does, another way is to use the factors to help you. Here,  $5 \times 3 = 15$  and  $10^3 \times 10^5 = 10^8$  (when multiplying exponents, simply add the exponents if the bases are the same), which gives you  $15 \times 10^8$ . However, the first factor of any number in scientific notation must be at least 1 but less than 10, so  $15 \times 10^8$  becomes  $1.5 \times 10^9$ .
12. E. Multiply the coefficients 5 and 6 to get 30. Then, multiply the base 10 and exponents, adding the exponents 4 and 4 to get  $10^8$ . Remember that the coefficient must be between 1 and 10, so we should express 30 as  $3.0 \times 10^1$ . Combined with the base and exponents of  $10^8$ , we would have an expression of  $3.0 \times 10^1 \times 10^8$ , which can be expressed as  $3.0 \times 10^9$ .
13. A. The product of the coefficients is  $3 \times 2 = 6$ . The product of the base and exponents is  $10^3 \times 10^2 = 10^5$ . Thus, we have  $6.0 \times 10^5$ .
14. E. To add or subtract numbers in scientific notation, convert them into standard form first, then stack. Here,  $9.5 \times 10^5$  is 950,000 and  $5.9 \times 10^4$  is 59,000. Stacking and adding results in 1,009,000, which converts back to  $1.009 \times 10^6$ . Choice H is equivalent, but not in scientific notation.
15. 7,560. To convert scientific notation into standard form, simply move the decimal point the number of places noted by the exponent. The exponent tells us to move the decimal point three places to the right, turning 7.56 into 7,560, which should be gridded in without the comma.
16. 0.68. The exponent tells us to move the decimal point back one place, turning 6.8 into 0.68.
17. 720.  $1.2 \times 10^{-3} = 0.0012$ , and  $6 \times 10^5 = 600,000$ , so their product is 720. Or, multiply  $1.2 \times 6$  and move the decimal over by two, since canceling exponential expressions with the same base leaves us with  $10^2$ .
18. 500.  $3 \times 10^5 = 30,000$ , and  $6 \times 10^2 = 600$ , so 30,000 divided by 600 = 500. When we perform operations on exponential expressions with the same term ( $10^5$  and  $10^2$ ), we are allowed to simply work with their exponents. Since we are dividing two exponential terms, we subtract the value of the exponents.  $5 - 2 = 3$ , so the exponential term becomes  $10^3$ .  $3 \div 6 = 0.5$  and  $0.5 \times 10^3 = 500$ .

19.  $3,960$ .  $4.6 \times 10^3 = 4,600$ , and  $6.4 \times 10^2 = 640$ , so the difference between the two distances is  $4,600 - 640 = 3,960$ .

## Counting Principle

1. B. We can list each different combination of different teams. The order is *not* important (that is, Alex and Bea is the same as Bea and Alex, so this would only count as one team). We'll abbreviate as follows: Alex = A, Bea = B, etc. So, the different combinations are AB, AC, AD, BC, BD, CD. There are 6 possible combinations.
2. G. Since there are only 3 different numbers (1, 2, and 3), the most efficient way to solve this is to list out all the possible 3-digit numbers: 11, 12, 13, 21, 22, 23, 31, 32.
3. B. There are different possible orientations of the red (R), yellow (Y), and blue (B) layers. From top to bottom, she could have RYB, RBY, YRB, YBR, BRY, or BYR.
4. F. If we imagine that each student is numbered from 1-5, with 1 being the shortest and 5 being the tallest student, we can list the different combinations. 1 will always be in the middle, so the other combinations will only affect the orientation of 2-5. Since 2 and 3 (the next taller students) will always be on the left of 1, we can imagine either 231 or 321. For each of these, to the right of 1 could be 45 or 54. So, we could have 23145, 23154, 32145, or 32154. This is a total of 4 different combinations.
5. C. Once the numbers in a question start to get higher, listing out every possible combination becomes too time consuming and prone to error. Instead, we can use what is known as the counting principle to help us solve this question. Each pair of pants can be paired with 10 different shirts, making 10 outfits. Since there are 4 different pairs of pants, then there are  $4 \times 10 = 40$  different outfits.
6. H. Since each of the 9 soups can be paired with each of the 6 salads, there are a total of  $9 \times 6 = 54$  possible combinations.
7. D. We can use the counting principle to solve this question. Since we know that Chet may repeat digits, each of the 3 characters can be any one of 10 possible digits. So,  $10 \times 10 \times 10 = 1,000$ . If the first digit is 0, the next digit could be 0, and the last could also be 0. This represents one combination of a code. We could cycle through every possible combination until the first digit is 9, the next digit is 9, and the last is also 9. In total, this represents 1,000 different combinations.
8. F. We can use the counting principle to solve this question. The password must have four letters, each of which can repeat from each of 26 letters. If we started at the very beginning of the alphabet, we would have a password of AAAA. By changing just one of these letters at a time, we would eventually cycle through every possible combination (AAAB, AAAC, etc.) until we arrived at a password of ZZZZ. This would be a total of  $26 \times 26 \times 26 \times 26 = 26^4$  different possible combinations.
9. D. There are  $3 \times 4 \times 3 = 36$  different possible sandwiches that can be made with the different types of bread, meat, and cheese.
10. G. Each question has four different possible answer choices. The answer to each of the questions could be all A's, or all D's, etc., or some other combination of A, B, C, and D. The answers could be filled in as AAA, AAB, AAC, etc., all the way up to DDD. Since this is the case, we have  $4 \times 4 \times 4 = 64$  different combinations.
11. A. Since the first 3 characters can be any letters from A-Z, each character can be any of the 26 letters (including the same letters). So for the first 3 characters, we have  $26 \times 26 \times 26 = 26^3$  possible combinations. The last 3 characters can be any of 10 digits (0-9), including the same digits. So for the last 3 characters, we have  $10 \times 10 \times 10 = 10^3$ . Since we must pair these combinations together into a 6 digit long combination, we must multiply both values together.



12. G. If Lewis were allowed to repeat digits, he would be able to choose from 10 digits (0-9) for each of the 3 characters in his code. This would mean  $10 \times 10 \times 10 = 1,000$  possibilities. However, he is not allowed to repeat digits in the 3 different character spaces. This means that though he can choose from any of the 10 digits for one character space, he can only choose from 9 digits for another character space and from 8 digits for another character space. This is because once a digit is chosen for a character space, it cannot be reused in another character space, thereby decreasing the total number of digits from which a number can be chosen. This means there are  $10 \times 9 \times 8 = 720$  possibilities.
13. B. In this question, the order each swimmer places in matters (gold, silver, then bronze going to Al, Bob, then Chet is different than Chet, Bob, then Al). For gold, there are 5 swimmers to choose from. Once one of the swimmers wins gold, there are only 4 swimmers left to give the silver to. Once that swimmer gets the silver, there are only 3 swimmers left to give the bronze to. Using the counting principle,  $5 \times 4 \times 3 = 60$ .
14. H. Since Gail cannot repeat letters in each of the four characters of her password, she can only choose from 26 letters in one of the character spaces, from 25 in another character space, from 24 in yet another character space, and 23 in the final character space. This is because once a letter is chosen for a character space, it cannot be reused in another character space, thereby decreasing the total number of letters from which a letter can be chosen.
15. D. For each of the first three characters, he may choose from the alphabet consisting of 26 letters. However, after he makes his first choice, he will not be allowed to repeat a letter. This means his second choice must be made from 25 letters, and his third choice from 24 letters. Thus, the total number of combinations is  $26 \times 25 \times 24$ . For each of the last three characters, he may choose from the digits 0-9, consisting of 10 different digits. However, as with the letters, once he makes a choice, he will have 1 fewer option to choose from. So, though his first choice will be from among 10 different digits, the second will be from 9 and the third from 8. Thus,  $10 \times 9 \times 8$ . Altogether, this makes  $26 \times 25 \times 24 \times 10 \times 9 \times 8$ .
16. F. Partha has 7 books to choose from for one of the positions above her desk. After she chooses that first book, it can't go in another one of the positions, so she only has 6 books to choose from. Once the second book is chosen, she only has 5 books to choose from for the last position above her desk. Therefore, she has  $7 \times 6 \times 5 = 210$  different ways to arrange the books above her desk.
17. 45. For each type of bread, there are 5 choices of meat, creating 15 sandwiches. Each of the 15 sandwiches get 3 choices of condiment, so the answer is  $3 \times 5 \times 3 = 45$ .
18. 720. Since 10 people are competing, there are 10 options for first place. Once one person places first, there are 9 options left for second place. Once one of those people places second, there are 8 options left for third place. The answer is  $10 \times 9 \times 8 = 720$ .
19. 24. To find all the possible outfit combinations, multiply  $3 \times 3 \times 3 = 27$ . However, we must remove the outfits that are all red, all yellow, and all blue, leaving us with 24 possible outfits.
20. 626. With 26 options for the first letter and 26 options for the second letter, there are  $26 \times 26 = 676$  different possible combinations, but 50 are already in use, so there are  $676 - 50 = 626$  options left.

### Imaginary Operations

1. B. We can substitute 4 for  $b$  in the equation, since the symbol  $\square$  appears before both  $b$  and 4. Doing so gives us  $6(4) - 12 = 12$ .
2. G. We must calculate the value of each  $\square 7$  and  $\square 3$  and subtract the latter from the former.  $\square 7 = 6(7) - 12 = 30$ . Similarly,  $\square 3 = 6(3) - 12 = 6$ . Therefore,  $30 - 6 = 24$ .
3. B. We are told that  $\square b = 42$ , therefore  $6b - 12 = 42$ . Simply solve for  $b$ :  $54 = 6b$ , and  $b = 9$ .



4. G. Substitute the value in between the two carrots (^ ^) for  $x$ . In this case, we get to  $2^2 + 6(2) + 9 = 25$ .
5. A. Substitute the value in between the two carrots (^ ^) for  $x$ . In this case, we get to  $-3^2 + 6(-3) + 9 = 0$ .
6. F. Set the equation equal to 64 and solve for  $x$ .  $64 = x^2 + 6x + 9$ . Simplify and then try each answer choice in the equation  $55 = x^2 + 6x$ . We see that 5 works, as  $55 = 25 + 30$ .
7. B. Substitute the values provided into the operation.  $x = 5$  and  $y = 3$ . Therefore,  $5@3 = 2(5) - 3(3)$ . This simplifies to  $10 - 9 = 1$ .
8. G. Substitute the value provided into the operation.  $y = 7$ . Therefore,  $x@7 = 2x - 3(7)$ . This simplifies to  $2x - 21$ .
9. B. Substitute the value provided into the operation.  $x = 7$ . Therefore,  $7@y = 2(7) - 3(y)$ . This simplifies to  $14 - 3y$ .
10. F. Substitute the value provided into the operation.  $c = 4$ . Therefore,  $4\#d = 4^2 + 4(d) + 16$ . This simplifies to  $32 + 4d$ .
11. D. Substitute the values provided into the operation. In this case, we are given the fact that  $c = 2$  and  $d = 3$ . Therefore,  $2\#3 = 2^2 + 2(3) + 16$ . This simplifies to  $4 + 6 + 16 = 26$ .
12. G. Substitute the values provided into the operation and solve each operation separately. In the first case, we are given the fact that  $c = 10$  and  $d = 4$ . Therefore,  $10\#4 = 10^2 + 10(4) + 16$ . This simplifies to  $100 + 40 + 16 = 156$ . In the second case, we are given the fact that  $c = 4$  and  $d = 10$ . Therefore,  $4\#10 = 4^2 + 4(10) + 16$ . This simplifies to  $16 + 40 + 16 = 72$ . The difference is then  $156 - 72 = 84$ .
13. C. Substitute the value provided into the operation. In this case, we are given the fact that  $p = -5$ . Therefore,  $-5\sim = \frac{1}{-5} + (-5)^2$ . This simplifies to  $25 - \frac{1}{5} = 24\frac{4}{5}$ .
14. E. Set the operation equal to 0, and we can solve for  $p$ .  $0 = \frac{1}{p} + p^2$ . Subtract  $p^2$  from both sides to get  $-p^2 = \frac{1}{p}$ . Multiply both sides by  $p$  to cancel the denominator:  $-p^3 = 1$ . Rid the variable of the negative by dividing both sides by  $-1$  to get  $p^3 = -1$ . The cube root of both sides gives us  $p = -1$ .
15. B. A fraction cannot have 0 as its denominator, as the result would be undefined (you can't divide something by nothing).
16. G. Substitute the value provided into the operation. In this case, we are given the fact that  $x = -2$ . Therefore,  $\ominus = \frac{-2}{4} + (-2)^2$ , which simplifies to  $4 - \frac{1}{2}$ , or 3.5.
17. D. Substitute the value provided into the operation. In this case, we are given the fact that  $x = 1$ . Therefore,  $\bigcirc = \frac{1}{4} + 1^2$ , which simplifies to  $1\frac{1}{4}$ , or  $\frac{5}{4}$ .
18. G. Set the operation equal to 17 and solve for  $x$ .  $17 = \frac{x}{4} + x^2$ . Since we have variables with different exponents, it is easiest to solve by trying each answer choice. By doing so, we find that 4 satisfies the equation, as  $\frac{4}{4} + 4^2 = 1 + 16 = 17$ .
19. 48. Plugging  $-2$  into the operation gives us  $(-2)^2 - 10(-2) + 24$ , which simplifies to  $4 + 20 + 24 = 48$ .

20. 13. We've been told that the result of the operation must be 100, we set up the equation  $9x - 7 = 110$  and solve for  $x$ , resulting in 13.
21. 8. Plugging in 10 and 2 gives us  $(10 + \frac{10}{4}) - (2 + \frac{10}{4})$ , which simplifies to  $12.5 - 4.5 = 8$ .
22. -19.2. Plugging 4 and 5 gives us  $\frac{4}{5} - 4(5)$ , which simplifies to  $0.8 - 20 = -19.2$ .
23. 9. Plugging in 6 for  $b$  and 63 for the result gives us  $5a + 3(6) = 63$ . Solving for  $a$  results in  $a = 9$ .
24. 5. Start inside the parentheses. Plug in 2 where you see  $q$ . This gives us  $2(2) - 1$ , or 3. The expression now becomes  $3\#$ , so we plug in 3 where we see  $q$ . This gives us  $2(3) - 1 = 5$ .

## Algebra

### Algebraic Expressions & Equations

1. D. If  $b = 3$ , then  $a$  must equal 12. Therefore,  $a^2 - 5b = 12^2 - 5(3) = 144 - 15 = 129$ .
2. G. To solve for  $x$ , start by simplifying the left side of the equation by combining like terms. If we subtract one-fifth of  $x$  from one  $x$ , we are left with fourth-fifths of  $x$ . The equation becomes  $\frac{4}{5}x = 8$ , which we can solve by dividing both sides of the equation by  $\frac{4}{5}$ . When we divide by a fraction, we actually multiply both sides by its reciprocal. This just means multiplying both sides by the "flipped" fraction, since doing so will cancel out the fraction on one side. In this case:  $\left(\frac{5}{4}\right)\left(\frac{4}{5}\right)x = \left(\frac{5}{4}\right)8$  gives  $x = \frac{40}{4} = 10$ .
3. B. To solve for  $x$ , start by distributing to get rid of the parentheses:  $4x - 5 + 3x = 16$ . Combining like terms gives us  $7x - 5 = 16$ . Adding 5 to both sides and then dividing both sides by 7 results in  $x = 3$ .
4. G. Removing the parentheses by distributing, we get  $3 - x - x + 3 = x$ . Combining like terms results in  $6 - 2x = x$ . Adding  $2x$  to both sides and then dividing both sides by 3 results in  $x = 2$ .
5. D. To begin solving any algebraic equation, distribute to remove parentheses. Here, you get  $10 + 3y + 8 = 4y - 2 + 9y$ . The next step is to combine like terms:  $3y + 18 = 13y - 2$ . Then we make sure the variable is only on one side of the equation by cancelling out the smaller variable:  $18 = 10y - 2$ . Finally, isolate the variable by cancelling out the constant and then dividing by the coefficient:  $20 = 10y$  and  $y = 2$ .
6. E. The most efficient way to solve is to simply plug in 6 for each  $x$  value and multiply. The answer choices become  $7 \times 9$ ,  $7 \times 5$ ,  $9 \times 3$ , and  $3 \times 7$ . The first option results in 63, which is the greatest of all the products.
7. A. Whenever we see a single equation with two different variables, we know that we will not be able to solve for each variable individually, but we won't need to. We want to recognize a relationship between the given equation and the expression we're trying to find. If we subtract  $y$  from both sides, we get  $2x + 2y = 0$ . The goal is to recognize that  $2x + 2y$  is a factor of  $12x + 12y$ . All we have to do is multiply both sides of the equation by 6, resulting in  $12x + 12y = 0$ .
8. G. Plugging in 5 for  $a$  gives us  $5x^2 + 5x + 5$ . Looking at the answer choices, we see that the goal is to correctly factor out 5 from the expression. If we factor 5 out of each term, we are left with  $5(x^2 + x + 1)$ . This matches only one answer choice exactly, and since there can only be one correct answer, we can eliminate all other answers.
9. C. To solve for  $x$ , we first want to cancel out the exponent by finding the square root of both sides of the equation. The square root of  $(x - 5)^2$  is  $x - 5$  because square roots cancel out squares. The square

root of 36 can be either 6 or  $-6$ , so we must solve for both options:  $x - 5 = 6$  results in  $x = 11$  while  $x - 5 = -6$  results in  $x = -1$ . Since the problem specifies that  $x$  must be less than 0, the answer is  $-1$ .

10. G. We should recognize that we can only solve for a variable when there is only one variable in the equation. We cannot solve for either variable in the first equation yet, so we will solve for  $x$  in the second equation. Combining like terms gives us  $11x = 11$ , so  $x = 1$ . Plugging that value into the first equation gives us  $1 = y - 9$ . Since there is now only one variable, we can solve for it: adding 9 to both sides results in  $y = 10$ .
11. B. Since we are asked to find the value of  $9x - 3$ , and since the equation  $9x + 7 = 11$  already contains the term  $9x$ , we need only to subtract 10 from both sides of the provided equation to arrive at  $9x - 3 = 1$ . We could solve for  $x$  using  $9x + 7 = 11$ , then plug that value into  $9x - 3$ , but this is time consuming. We would first subtract 7 from both sides, then divide by 9. This would result in  $\frac{4}{9}$ , which we would plug into the  $x$  value of  $9x - 3$  to arrive at  $4 - 3 = 1$ .
12. G. The  $@$  symbol is a variable, just like  $x$  or  $y$ . To solve for  $@$ , we want to cancel out the other numbers on that side of the equation, namely the 5 and the 2. Because 5 and  $@$  are both being divided by 2, we'll cancel out the 2 first by multiplying both sides of the equation by 2, leaving us with  $5 + @ = 11$ . Now simply subtract 5 from both sides, leaving us with  $@ = 6$ .
13. C. Create an equation to represent the information given. "Twice a number" can be represented as  $2x$ . If 8 more than this is equal to 14, then it can be expressed as  $2x + 8 = 14$ . Solve for  $x$  by subtracting 8 from both sides then dividing by 2 to get  $2x = 6$  and then  $x = 3$ . Remember: the question is not asking us to find this number. We are trying to find 8 times the number, which is  $8 \times 3 = 24$ .
14. H. Express the information provided algebraically. Let  $x$  represent the unknown number. The first part of the question tells us that  $\frac{2}{3}x = 32$ . Divide both sides of the equation by  $\frac{2}{3}$ ; this is the same as multiplying both sides by the reciprocal, or  $\frac{3}{2}$ . So,  $32 \times \frac{3}{2} = 48 = x$ . We are asked to find  $\frac{3}{4}$  of  $x$ , which is  $48 \times \frac{3}{4} = 36$ .
15. D. There are several ways to solve this question. Algebraically, we can solve for  $a$  using  $\frac{3}{5}a = 30$  and substitute the value of  $a$  into  $\frac{4}{5}a$  to solve the question. Under this method, we would first multiply both sides of the equation by the reciprocal of the fraction:  $\left(\frac{5}{3}\right)\left(\frac{3}{5}a\right) = \left(\frac{5}{3}\right)(30)$ . This leaves us with  $a = 50$ . Then, substitute into  $\frac{4}{5}a$ :  $\left(\frac{4}{5}\right)(50) = 40$ . This is perfectly valid, but time consuming. We can intuit that because  $\frac{3}{5}$  of  $a$  is equal to 30, then each  $\frac{1}{5}$  portion of  $a$  must equal 10. Four  $\frac{1}{5}$  portions of  $a$  must, then, equal 40.
16. H. We can set up an algebraic equation to represent the information provided.  $c$  multiplied by 5 can be represented as  $5c$ . 5 added to  $c$  can be represented as  $c + 5$ . If they are "the same," then they must be set equal to one another. Thus,  $5c = c + 5$ . Simplify by subtracting  $c$  from each side:  $4c = 5$ . We could simplify further, but we don't need to: we're asked to find  $4c$ , and the equation already tells us  $4c = 5$ .

17. B. When dividing a whole number by a fraction, we simply multiply the numerator by the reciprocal of the denominator (the fraction) and simplify. We can look at each term individually.  $\frac{1}{1/2} = 1 \times \frac{2}{1} = 2$ . The next term can be expressed as follows:  $\frac{1}{\left(\frac{1}{2}-1\right)} = \frac{1}{\left(-\frac{1}{2}\right)} = 1 \times -\frac{2}{1} = -2$ . Therefore,  $2 + (-2) = 0$ .
18. G. We have been given the value for  $c$ , so we can just work our way backwards through each statement to solve for  $a$ .  
 $b = 4(3) = 12$ . Substitute 12 in for  $b$  in the first statement:  $a - 12 = 10$ . Add 12 to both sides;  $a = 22$ .
19. D. Simply if the expression by multiplying:  $2 \times \frac{m}{n} \times n^2 = \frac{2mn^2}{n}$ . Then simplify by cancelling one  $n$  from the numerator and denominator, leaving us with  $2mn$ . That is all we need to do. The problem is asking for the value of  $2mn$ , which is simply twice the value of  $mn$ . Since  $mn = 10$ , we know that  $2mn = 20$ .
20. G. To solve any algebraic equation that consists of two fractions set equal to each other, start by cross multiplying. Here, you get  $0.16 \times x = 0.40 \times 2.5$ , which simplifies to  $0.16x = 1$ . To isolate the variable, divide both sides by 0.16, which results in  $x = 6.25$ .
21. A. Concentrate on each statement separately. First, “the sum of  $x$  and the square root of  $y$ ”: this can be represented by  $x + \sqrt{y}$ . “Is equal to” is simply an “=” sign. “The square root of the sum of  $x$  and  $y$ ,” can be represented by  $\sqrt{x+y}$ . Thus,  $x + \sqrt{y} = \sqrt{x+y}$ .
22. G. As with other problems, this question tests students’ abilities to figure out a function based on a numerical pattern. If we assign a variable,  $n$ , to any position in the sequence, we see that the value of the position can be multiplied by 8 and have 1 subtracted from it to produce the number in that position. Therefore, plugging in 25 will produce the 25<sup>th</sup> value in the sequence.
23. C. To solve for the variable we want to first cancel out the 38. If we divide both sides of the equation by 38, we are left with  $101 = x + 1$ , so  $x = 100$ .
24. G. The problem tells us that the list of values is made exclusively of multiples of 3, which we can see by looking down the list. However, we cannot simply multiply the Position number by 3. The 1,000<sup>th</sup> value will not be 3,000 because that would require starting the list with  $3 \times 1$ . Instead, every Value has 3 subtracted from it, so the 1,000<sup>th</sup> value will be 2,997.
25. C. Whenever we see parentheses in an equation, we want to think about whether we should start by distributing. Here, distributing would result in  $ax + bx = 28$ . The problem tells us what  $ax$  equals, and asks us to find  $bx$ , so this was the correct first step. We can now plug in 7 for  $ax$ , giving us  $7 + bx = 28$ . Now all we have to do is subtract 7 from both sides, leaving us with  $bx = 21$ . We do not have to find the value of  $b$  or  $x$  individually.
26. H. If  $s$  represents the smallest of three consecutive even integers, then the other two integers are  $s + 2$  and  $s + 4$ . The sum would be  $s + s + 2 + s + 4$ , which simplifies to  $3s + 6$ .
27. D. We can solve any proportion by cross multiplying, but here we can also solve by recognizing that the denominator of the left-hand fraction is 4 less than the numerator. Likewise, in the right-hand fraction, 39 is 4 less than 43. Therefore,  $x$  must also be 43 for the two sides to be equal.
28. G. We can solve by cancelling out the 1,000 using division, leaving us with  $73 = 7p + 3$ , which results in  $70 = 7p$  and finally  $p = 10$ .
29. C. Regardless of how many variables an equation has, we solve for the target variable by isolating it. Here, the questions is asking us to isolate the  $b$ , which we do by subtracting  $5a$ , resulting in  $b = a + 5 - 5a$ . Combining like terms gives us  $b = 5 - 4a$ .



30. H. To translate an equation from words to symbols, first find where the equal sign goes, represented by “is equal to”. Now look at the words to the left, “the product of  $x$  and  $\frac{2}{5}$ ,” which becomes  $\frac{2}{5}x$ , and the words to the right, “the sum of  $5x$  and  $2$ ” becomes  $5x + 2$ .
31. 5. We know what  $y$  equals, but we don’t know yet what  $x$  equals, so we plug 3 in for  $y$  in the first equation to solve for  $x$ :  $\frac{x}{3^2} = 5$ . This simplifies to  $\frac{x}{9} = 5$ , which results in  $x = 45$ . Now we can plug in 45 and 3 into the final equation:  $\frac{45}{3+2(3)} = \frac{45}{9} = 5$ .
32. 144. We don’t know what  $a$  or  $b$  equals, only that their product is 12, but that is all we need to know.  $4a \times 3b$  equals  $12ab$ . If we plug 12 in for  $ab$ , we get  $12 \times 12 = 144$ .
33.  $-5$ . Substituting 4 into the equation gives us  $a^2 - 16 = 9$ . Combine like terms for  $a^2 = 25$ , and solve for  $a = 5$  or  $a = -5$ . Since  $b = 4$  and  $a < b$ , we must choose  $-5$ .
34. 34. The sentence describes an algebraic equation:  $2(2x - 12) = 10 - x$ , which can be simplified to  $4x - 24 = 10 - x$ . Adding  $x$  and 24 to both sides results in  $5x = 34$ . We don’t need to solve for  $x$  because we are only asked to find the value of  $5x$ .
35. 2. To solve a proportion, cross multiply. That give us  $3(x + 30) = 8(x + 10)$ , which simplifies to  $3x + 90 = 8x + 80$ . Solving for  $x$  results in  $x = 2$ .

### Algebra in Context

- C. Let  $x$  = the number of books that Malcolm has. In terms of  $x$ , Yannis has  $x + 8$  books. Together, they have 60 books. This can be represented as:  $60 = x + x + 8$ , or  $60 = 2x + 8$ . Solve for  $x$ . Subtract 8 from both sides:  $52 = 2x$ . Divide both sides by 2:  $x = 26$ . This represents Malcolm’s books. Yannis has 8 more, or  $26 + 8 = 34$ .
- G. Let  $x$  = Leon’s age. In terms of  $x$ , Michael’s age can be represented as  $2x + 3$ . Since we know Michael is 35, we set the equation for Michael’s age equal to 35:  $35 = 2x + 3$ . Solve for  $x$  by subtracting both sides by 3 then dividing both sides by 2. So,  $x = 16$ .
- C. Let  $x$  = the number of bumpers a human can make. A machine can therefore make  $10x$  bumpers. Altogether, they can make  $x + 10x$ , or  $11x$ , bumpers. Since we know the total is 550, we can set up the equation  $11x = 550$ , so  $x = 50$ . But that number represents the number of bumpers a human can make. The question asks for the number a machine can make, which is  $50(10) = 500$ .
- F. Let  $x$  = Tom’s pages, in which case  $2x + 12$  equals Sue’s pages. We know Sue read 90 pages, so  $2x + 12 = 90$ , in which case  $x = 39$ , which represents Tom’s pages.
- C. We know that  $C$  is a right angle, so the sum of the degree measures of angles  $A$  and  $B$  is 90. If we let  $x$  = the degree measure of angle  $A$ , then angle  $B = 2x - 18$ , in which case their sum is  $3x - 18$ . We can set up the equation  $3x - 18 = 90$ , in which case  $x = 36$ . This is the degree measure of angle  $A$ , so angle  $B$  must be  $2(36) - 18 = 54$ .
- G. The total degree measure of a triangle is 180, so the three angles of the given triangle can be represented as  $x + 2x + 3x = 180$ , which can be simplified to  $6x = 180$ , in which case  $x = 30$ . We are looking for the largest angle, which is  $3x = 3(30) = 90$ .
- C. The total degree measure of a triangle is 180, so the three angles of the given triangle can be represented as  $3x + 4x + 5x = 180$ , which can be simplified to  $12x = 180$ , in which case  $x = 15$ . We are looking for the smallest angle, which is  $3x = 3(15) = 45$ .

8. H. Let  $x$  = Ronan's current age, in which case  $\frac{1}{3}x$  will be Shonda's current age. The second sentence describes the equation:  $\frac{1}{3}x + 10 = 0.5(x + 10)$ . Using standard algebra we get  $x = 30$ .
9. A. You can set up an algebraic equation to solve, but you don't need to. The two machines together will make 150 sneakers per minute. To find out how many minutes it will take them to make 5,400 sneakers, divide 5,400 by 150, which results in 36.
10. G. The variable  $c$  represents the number of cars Sherman has washed, for which he is paid \$4 each, so the amount of money he makes is  $4c$ . To that we add the flat rate of \$60, which is not multiplied by  $c$  because he does not earn \$60 for each car he washes.
11. B. We can solve this question by substituting values of  $t$  into the equation provided.  $p(t) = 5,000 \times 2^{\left(\frac{3}{3}\right)}$  is really just
- $p(t) = 5,000 \times 2^1$ , which is  $5,000 \times 2 = 10,000$ .  $p(t) = 5,000 \times 2^{\left(\frac{9}{3}\right)}$  simplifies to  $p(t) = 5,000 \times 2^3$ , which is  $5,000 \times 8 = 40,000$ . The difference is 30,000.
12. G. The phone company will charge 12 cents for every minute except the first minute because they charged 30 cents for that first minute. That means there would be a flat rate of \$0.30 and then \$0.12 for each minute minus the first minute, or  $0.30 + 0.12(m - 1)$ .
13. 6. If a door is 10 feet long, and this is 4 feet longer than the width, the width must be  $10 - 4 = 6$ .
14. 45. If we let  $x$  = the number of pushups Luke's brother can do, then Luke can do  $3x$ , and we can set up the equation  $x + 3x = 60$ , which simplifies to  $4x = 60$ , resulting in  $x = 15$ , so Luke can do  $3(15) = 45$  pushups.
15. 18. To solve, set up an equation. Let  $x$  = Ed's age. Pam's age is therefore  $2x - 10$ . If the sum of their ages is 32, then  $x + 2x - 10 = 32$ . Solving for  $x$  results in  $x = 14$ , which is Ed's age. Pam's age is  $2(14) - 10 = 18$ .
16. 26. If we assign  $x$  for each of the two legs, we can set up the equation  $x + x + 2x - 6 = 46$ , which simplifies to  $4x - 6 = 46$ . Solving for  $x$  results in  $x = 13$ , so the sum of the two congruent legs is 26.
17. 150. The total degree measure of the 4 angles of any quadrilateral is 360, so the relationship can be represented as  $x + 2x + 4x + 5x = 360$ , which simplifies to  $12x = 360$ . Solving for  $x$  results in  $x = 30$ , so the largest angle is  $5(30) = 150$ .
18. 17. We can assign  $x$  for the number of nickels James has, so he has  $x + 8$  dimes. The value of his coins will then be  $0.05x$  and  $0.10(x + 8)$ , and we can set up the equation  $0.05x + 0.10(x + 8) = 2.15$ , which can be simplified to  $0.05x + 0.10x + 0.80 = 2.15$ . Combining like terms gives us  $0.15x = 1.35$ , which results in  $x = 9$ . Therefore, he has 9 nickels and 17 dimes.
19. 6. The word "is" denotes an equal sign, so  $24 - 3x = 5x + 8$ . Combine like terms for  $16 = 8x$ . Divide both sides by 8 for  $x = 2$ . The problem is asking for  $3x$ , which is  $3(2)$ , or 6.
20. 11. The base cost for TubeMusic is \$8.75, which Harrison would incur no matter how many songs he downloaded. This means he spent  $\$17 - \$8.75 = \$8.25$  on downloading songs. This is  $\$8.25 \div \$0.75 = 11$  songs.
21. 0. Start by finding  $y$ :  $7y + 21 = 14$ . Combine like terms for  $7y = -7$ . Divide both sides by 7 to isolate the variable for  $y = -1$ . Then, multiply  $y$  by 3 for  $-3$ . Add this to 3 for 0.
22. 8. The normal cost of cherries is \$8. If cherries are on sale for 25% off, then they cost only \$6 (since  $8 \times 0.75 = 6$ ). Create an equation for the cost of cherries: pounds of cherries  $\times$  price per pound = total cost. If we know we only have \$20, then this is the total cost of the cherries. So, we have the following: pounds of cherries  $\times 6 = 20$ . Divide both sides by 6 for 8 pounds total.

23. 30. The total number of hours worked can be expressed as follows: hours worked by Rai + hours worked by Caleb. Let's represent the former by  $r$  and the latter by  $c$ . So, we know  $r + c = 73$ . We know that Caleb worked 13 more hours than Rai. So, if the number of hours Rai worked is  $r$ , then we know  $c = r + 13$ . Substitute this into the original equation for  $r + 13 + r = 73$ . Combine like terms for  $2r = 60$ , and  $r = 30$ .

## Plugins

1. A. If  $x < 0$ , it is a negative number. Any negative number multiplied by a positive number will also be a negative number. The smaller the negative number (the less negative it is), the greater the value it has. Thus,  $x$  will be greater than any of the other answer choices so long as  $x$  is negative.
2. H. If  $m$  represents an odd integer, then  $m + 2$  also represents an odd integer.  $2m$  represents an even integer, but  $2m - 1$  represents an odd integer.  $3m$  represents an odd integer, and  $3m - 2$  and  $3m + 2$  both also represent odd integers.  $5m$  represents an odd integer, so  $5m + 1$  represents an even integer. We can also solve this question by plugging in an easy-to-work-with odd number, like 1.
3. B. The problem is asking for the value of  $4b$ . This means we should solve for  $b$  first. Multiply both sides of the equation by 5, resulting in  $b = 5a$ . To then find the value of  $4b$ , multiply both sides by 4, resulting in  $4b = 20a$ .
4. F. Multiplying any integer by  $10^{-3}$  means to simply move the decimal point three places to the left. Since there is no decimal shown in ABC, that means it is at the end (as in ABC.0). Therefore,  $ABC \times 10^{-3} = 0.ABC$ .
5. D.  $a + b < a$  means that we're taking some number  $a$ , adding another number  $b$  to it, and the answer is somehow less than the original number  $a$  that we started with. How is that possible?  $b$  has to be a negative number!
6. G. Since  $c$  equals both  $ab$  and  $kb$ , that means that  $ab = kb$ . Since  $b$  is the same in both equations,  $k$  must equal  $a$ .
7. B. If  $x$  represents the number of years Mrs. Simmons has been teaching, then twice that number of years would be  $2x$ . Mr. Thompson has been teaching 8 years less twice Mrs. Simmons, which would be  $2x - 8$ .
8. F. If 1 is added to  $a$ , which is an even integer, we will end up with an odd integer. For example, if  $a = 2$ , then  $2 + 1 = 3$ , which is odd. To get to the next even integer, we must add 2.
9. C. We can substitute values in for  $a$ ,  $b$ , and  $c$  to see which answer choice is true.  $a$  must be greater than  $b$ , so we can let  $a = 2$  and  $b = 1$ . Now we can add any number  $c$  – let's choose 0 for  $c$  – to both numbers.  $a + c = 2$  and  $b + c = 1$ . The difference between these two sums is 1, which tells us that the sum of  $c$  and  $a$  is 1 greater than the sum of  $c$  and  $b$ . Then, we can plug in the same values for  $a$ ,  $b$ , and  $c$  into the answer choices and see which choice gives us 1. Only  $a - b$  gives us the same answer.
10. H. There are a few simple rules to help with these questions. The addition or subtraction of two even or two odd integers will always result in an even integer; the addition or subtraction of one even and one odd integer will always result in an odd integer. With multiplication, *only* when two odd numbers are multiplied will the result be odd; in all other circumstances, the multiplication of two integers will always result in an even number. In this case, increasing or decreasing  $x$  by 1 results in an even number.
11. C. There are two rates described in this question: a per day fee (\$1.30) as well as a per minute fee (\$0.13). To calculate the cost having this phone plan, we need to know two variables: the number of days, and the number of minutes. We are told to assume that the plan is active for one day, but are only told that the phone is used for  $x$  minutes of talking. Thus,  $\$1.30 \text{ per day} \times 1 \text{ day} + \$0.13 \times x$ , which simplifies to  $1.30 + 0.13x$ .

12. H. Substitute 1 into  $q$  for  $1 - \frac{1}{4}p = 0$ . Subtract 1 from both sides for  $-\frac{1}{4}p = -1$ . Divide both sides by  $-\frac{1}{4}$  for  $p = 4$ . Prove this by resubstituting  $p$  into the equation.
13. C. The shape that's being described has 5 lengths, four of which are  $x$  and one of which is  $y$ . The means the total length of tape should be  $4x + y$ . However, none of the answer choices have  $y$ . Since the area of a rectangle is length times width, and we know that the area is 600, we can create the equation  $xy = 600$ , in which case  $y$  can be written as  $\frac{600}{x}$ , and the total length of tape is now  $4x + \frac{600}{x}$ . That is still not an answer choice, but if we simplify the fractions in the choices, we see that C simplifies to  $4x + \frac{600}{x}$ .
14. H. The first book costs  $b$  dollars. The remaining 4 books each cost  $b - d$  dollars. The total amount would then be  $b + 4(b - d)$ , which simplifies to  $b + 4b - 4d$ , which equals  $5b - 4d$ .
15. B. We can substitute  $6c$  in for  $5b$  in the first equation and arrive at  $2a = 6c$ . Simplifying,  $a = 3c$ .
16. E. The first ticket is  $m$  dollars. Each ticket after that is  $m - n$  dollars. We cannot multiply the total number of tickets  $x$  by  $m - n$  because the first ticket was not at the rate; we can instead multiply  $m - n$  by  $x - 1$ , and add that to the price of the first ticket, which was  $m$ .
17. D. The percentage of something can be expressed as "the part"  $\div$  "the whole." In this case, we know that  $b$  is the part: the number of boys enrolled at the school. To determine "the whole" (the total number of boys and girls enrolled at the school), we must use the information provided. We already know that  $b$  represents the number of boys. In terms of  $b$ , there are  $1.25b$  girls enrolled at the school; this represents the fact that there are 25% more girls than boys at the school, since we multiply whatever the number of boys is by 1.25 to get the number of girls. The denominator becomes  $2.25b$ , so the percent of the school that is boys is  $\frac{b}{2.25b} \times 100$ .
18. E.  $g$  represents the total number of eggs. We are told that all of the crates,  $c$ , are filled except for one, which had 5 empty slots. Each crate holds  $m$  eggs when filled. So, if there are  $c$  crates, each filled to  $m$  capacity except for 5 slots, then  $g = c \times m - 5$ .
19. D. Choose any two consecutive odd integers – say, 5 and 7. Plugging them into the expression  $x^2 - y^2$  gives us  $7^2 - 5^2 = 49 - 25 = 24$ . All of the answer choices are in terms of the smaller of the two integers,  $y$ . Plugging 5 into each choice reveals that only one gives 24.
20. H. If the average of  $a$  and  $b$  is  $z$ , then  $a + b = 2z$ . Therefore, the average of  $a$ ,  $b$ , and  $c$  equals  $\frac{2z + c}{3}$ .
21. B. We can see that simply multiplying  $m + 5$  by 2 results in  $2m + 10$ . What is done to one side of an equation must be done to the other. Thus,  $2m + 10 = 2z$ .
22. H. If Kim was  $n$  years old  $m$  years ago, then she is  $n + m$  years old now, so 1 year ago she must have been  $n + m - 1$ .
23. C. We can use substitution to help us find the correct answer choice. Since  $x$  can't be equal to 0, we can use any other number  $x$  to be the first term. Let  $x = 2$ . We know that the next term in the sequence must be 2 more than 0.5 of the first term. This is  $2(0.5) + 2 = 3$ , so the second term is 3. The ratio of the second term to the first term, then, is  $3:2$ , or 1.5. We can substitute 2 in for  $x$  in the answer choices to see which gives us 1.5.
24. E. Plug in 9 for  $n$  to get  $(m + 2)^2 - 9 = 0$ . Isolate  $m$  first by adding 9 to both sides, giving us  $(m + 2)^2 = 9$ . Take the square root for  $m + 2 = 3$ . Subtract 2 for  $m = 1$ .
25. B. Using substitution can help us solve this question. We can use two easy-to-work-with numbers like 4 and 5, since they differ by 1. The sum of these two numbers is 9, so  $x = 9$ . We can plug in 9



for each instance of  $x$  in the answer choices and know that we want to end up with 5 (since that is the greater of the two numbers. Only

$$(9 + 1) \div 2 = 5.$$

26. E. Pick any number for  $x$  – say, 20. That means each ounce costs 2 dollars. If we choose 2 for  $y$ , then 2 dollars' worth of tea leaves will make 2 cups of tea, so 1 cup of brewed tea will cost \$1. Plugging 20 and 2 into each answer choice for  $x$  and  $y$ , we see that only this choice results in a value of 1.
27. A. The question tells us that the spring's length is given by the equation  $l = 12 + 2.5m$ . Since we know that  $l = 27$ , plug in:  $27 = 12 + 2.5m$ . Subtract 12 from both sides for  $15 = 2.5m$ . Divide both sides by 2.5 for  $m = 6$ .
28. G. When answer choices have variables in them, you can plug in numbers for each variable to see which choice works. Just remember to pick numbers that are factors or multiples of the numbers given in the problem. Here, we might say that the store sells 70 bottles per week, and each bottle costs 2 dollars. That means it earns \$140 per week, which we can divide by 7 to get \$20 per day. If we plug in 70 for  $b$  and 2 for  $d$ , only  $\frac{bd}{7}$  results in 20.
29. D. Pick a number for the total cost of the trip – say, \$100 – as well as a number for the total number of students in the class – say, 10 – and the number of students not going – say, 9. If everyone in the class went, each student would have to pay \$10. With 9 students not going, that leaves only 1 student who is going, which means each student must now pay a total of \$100, which is an additional \$90. Plugging these values into each answer choice, only the last choice results in 90.
30. F. Pick any two integers for  $x$  and  $y$  that have a difference of 4. Let's try 10 and 14. The average of 10 and 14 is 12. If we plug in 10 and 14 for  $x$  and  $y$  in each answer choice, only  $y - 2$  results in 12.

## Inequalities

1. A. To solve an inequality, use the same rules as when solving algebraic equations. First cancel out the constant by subtracting 4 from both sides, resulting in  $2x < 26$ . Then cancel out the coefficient by dividing both sides by 2, resulting in  $x < 13$ .
2. H. First cancel out the constant by subtracting 6 from both sides, resulting in  $-2x < 22$ . Then cancel out the coefficient by dividing both sides by  $-2$ , resulting in  $x > -11$ . \*Remember that, when working with inequalities, dividing both sides by a negative number causes the inequality to switch directions!
3. B. First cancel out the constant by subtracting 8 from both sides, resulting in  $1 > 7x$ . Then cancel out the coefficient by dividing both sides by 7, resulting in  $\frac{1}{7} > x$ , or  $x < \frac{1}{7}$ .
4. F. First cancel out the constant by subtracting 8 from both sides, resulting in  $3 > -5x$ . Then cancel out the coefficient by dividing both sides by  $-5$ , resulting in  $-\frac{3}{5} < x$ , or  $x > -\frac{3}{5}$ . \*Remember that, when working with inequalities, dividing both sides by a negative number causes the inequality to switch directions!
5. A. This problem is not asking us to find all possible values of  $z$ , but rather to find which is the only one of the given four choices that is a possible value for  $z$ . One way is to solve the inequality algebraically (resulting in  $z < -\frac{10}{3}$ ) and then see which answer choice falls within the given range (only  $-4$  is less than  $-\frac{10}{3}$ ). Another way to solve is to plug each answer choice into the given inequality and see which one results in a correct statement.
6. H. This problem is not asking us to find all possible values of  $m$ , but rather to find which is the only one of the given four choices that is not a possible value for  $m$ . One way is to solve the inequality

algebraically (resulting in  $m < 3$ ) and then see which answer choice does not fall within the given range (3 is the only value that is not less than 3). Another way to solve is to plug each answer choice into the given inequality and see which one results in a false statement.

7. D. We can distribute the  $-2$ , but we can also just cancel out the  $-2$  by dividing both sides of the inequality by  $-2$ , giving us  $x - 6 > -9$ . (\*Remember that, when working with inequalities, dividing both sides by a negative number causes the inequality to switch directions!) Lastly, cancel out the constant by adding 6 to both sides, resulting in  $x > -3$ .
8. G. We only want the variable on one side of the inequality, so we will cancel out the smaller variable by subtracting  $3x$  from both sides, giving us  $16 > 2x - 28$ . Now we can isolate the variable by adding 28 to both sides, giving us  $44 > 2x$ , and then dividing both sides by 2, resulting in  $22 > x$ , or  $x < 22$ .
9. B. We can distribute the negative sign, but we can also just cancel it out by dividing both sides of the inequality by  $-1$ , giving us  $7x + 8 \geq -9$ . (\*Remember that, when working with inequalities, dividing both sides by a negative number causes the inequality to switch directions!). Subtract 8 from both sides, giving us  $7x \geq -17$ , and then divide both sides by 7, resulting in  $x \geq -\frac{17}{7}$ .
10. G. We only want the variable on one side of the inequality, so we will cancel out the smaller variable by adding  $12x$  to both sides, giving us  $-46 \geq 33x + 64$ . Now we can isolate the variable by subtracting 64 from both sides, giving us  $-110 \geq 33x$ , and then dividing both sides by 33, resulting in  $-\frac{10}{3} \geq x$ , or  $x \leq -\frac{10}{3}$ .
11. C. If all riders must be at least 54 inches tall, then any value that is 54 or larger satisfies the inequality.  $r > 54$  does not work because it says that a rider's height must be greater than or equal to 54.
12. F. The heights must all be greater than or equal to 44 because the shortest student is 44 inches tall. The heights must all be less than or equal to 72 because the tallest student is 72 inches tall. Only this choice satisfies both those requirements.
13. B. The expression starts with 12. The words "is greater than" translates to the greater than symbol, or  $>$ . "Five less than twice a number  $x$ " becomes  $2x - 5$  (not  $5 - 2x$ ) because whatever value precedes "less than" is the value to be subtracted.
14. E. " $x$  is less than or equal to twice the value of  $y$ " can be written as  $x \leq 2y$ . " $y$  is less than zero" can be written as  $y < 0$ . We need a matching variable, so we can multiply both sides by 2 to get  $2y < 0$ , in which case we can create the compound inequality  $x \leq 2y < 0$ .
15. A. Begin by solving for  $q$ . Subtract 12 from both sides for  $-3q^2 < -12$ . Divide both sides by  $-3$  for  $q^2 > 4$  (remember to flip the inequality!). The only number which, when squared, gives a result greater than 4 is  $-3$ .
16. H.  $x \geq 4$  means any number 4 or larger satisfies the inequality so we shade in the number line above the 4. We want the circle over the 4 to be filled in because 4 itself also satisfies the inequality.
17. A.  $a < -6$  means any number less than  $-6$  satisfies the inequality so we shade in the number line to the left of  $-6$ . We want the circle over the  $-6$  to be open because  $-6$  itself does not satisfy the inequality.
18. F. To find the correct answer, we first need to solve the inequality algebraically. Subtracting 3 from both sides and then dividing both sides by 3, we get  $x \geq 1$ . That means we want a closed circle over 1 with a number line shaded to the right.
19. C. To solve the inequality algebraically, we subtract 5 from both sides, giving us  $-3a \geq -12$ , and then divide both sides by  $-3$ , resulting in  $a \leq 4$ . (\*Remember that when you multiply or divide by a negative number, the inequality switches direction!) That means we want a closed circle over 4 with the number line shaded to the left.

20. E. To solve this compound inequality algebraically, we want to isolate the variable by subtracting 3 from all three sides, giving us  $-12 \leq 3h \leq 3$ , and then dividing all three sides by 3, resulting in  $-4 \leq h \leq 1$ . That means that  $h$  can be any value greater than or equal to  $-4$  and less than or equal to 1. We show this on a graph with closed circles over  $-4$  and 1 and the number line shaded in between them.
21. 19.  $|x - 68| \leq 9$  can also be written as  $-9 \leq x - 68 \leq 9$  and simplified to  $59 \leq x \leq 77$ . This means the low temperature was 59 and the high temperature was 77. There are 19 integers from 59 to 77, inclusive.
22. 3.  $|x + 4| < 8$  can be written as  $-8 < x + 4 < 8$ , in which case  $-12 < x < 4$ .  $|x - 5| < 3$  can be written as  $-3 < x - 5 < 3$ , in which case  $2 < x < 8$ . The only integer that satisfies both inequalities is 3.
23.  $-4$ . First subtract 9 from all sides, giving us  $6 < -2x < 10$ . Then divide all sides by  $-2$ , giving us  $-3 > x > -5$ , or  $-5 < x < -3$ . There is only one integer that falls within this range, as  $-3$  and  $-5$  are both omitted.
24. 7. We can plug 6 in for  $x$ , giving us  $5(6) - 4y < 3$ , which simplifies to  $30 - 4y < 3$ . Solving for  $y$  results in  $y > \frac{27}{4}$ . The least integer value that satisfies this is 7.
25. 84. If books sell for 10 dollars each, then the total revenue could be called  $10x$ . If revenue must exceed production cost, then  $10x > 500 + 4x$ . Solving for  $x$  results in  $x > \frac{500}{6}$ , or  $x > 83.3$ . Therefore, 84 is the least number of books that must be sold.

## Probability & Statistics

### Probability

1. C. The probability of some specific event happening can be represented as a fraction or a percentage. In this case, the “whole” is the total number of marbles in the bag (12). This will be the denominator in our fraction. The numerator will be the number of white marbles in the bag (4), since we “want” to draw a white marble. Therefore, the probability of the thing that we “want” to happen (to draw a white marble) can be represented as  $\frac{4}{12}$ , which simplifies to  $\frac{1}{3}$ .
2. F. Since the plastic cylinder contains 12 dice, and 5 are red and 4 are white, we know that there must be 3 blue dice in the cylinder. Since there are 3 chances to draw a blue dice out of 12 possible dice, the probability is  $\frac{3}{12} = \frac{1}{4}$ .
3. A. The normal work day lasts for 8 hours (9:00 [9:00 a.m.] – 17:00 [5:00 p.m.]). The three 10-minute coffee breaks lasts for a total of 30 minutes, or 0.5 hours. Since the thing that we are trying to determine is the chance that the office worker will be on break, we can divide 0.5 hours (the total time during the day spent on break) by the total number of hours in the work day (8). This gives us  $\frac{0.5}{8} = \frac{1}{16}$ .
4. G. During any given 30-minute interval, we are told that there are 8 minutes of commercials. The question asks us to determine the chance that a commercial will be airing when the television is tuned to the station in question. This means we “want” the commercials to happen, so the numerator of our fraction is 8. The denominator is the total number of minutes, or 30.  $\frac{8}{30} = \frac{4}{15}$ .

5. B. There are a total of 3 hours between 8-11 p.m., or 180 minutes. The commercial is 1.5 minutes long, and will be played 4 times, for total air time of  $1.5 \times 4 = 6$  minutes. 6 minutes out of a possible 180 minutes is  $\frac{6}{180} = \frac{1}{30}$ .
6. G. Since it is not possible to choose a part or piece of a lollipop, the total number of lollipops must be a multiple of 4. 49 is not evenly divisible by 4 (it will result in a remainder, or partial piece of a lollipop). If there were 49 lollipops in the bucket, then there would be  $3 \times 49 \div 4 = 36.75$  grape-flavored lollipops in the bucket.
7. C. Let  $x$  represent the area of one of classrooms 1, 2, and 3. The total area of classrooms 1, 2, and 3 can be represented as  $x + x + x = 3x$ . In terms of  $x$ , the area of one of classrooms 4, 5, and 6 is  $2x$ . The total area of classrooms 4, 5, and 6 can be represented as  $2x + 2x + 2x = 6x$ . The total area of all 6 classrooms can therefore be represented as  $3x + 6x = 9x$ . The thing that we “want” is to find out the chance of randomly pointing at classroom 4, which has area  $2x$ . Thus, the probability is  $\frac{2x}{9x} = \frac{2}{9}$ .
8. F. Since the probabilities of selecting an orange and green candy are given, we know that the total number of candies must be a multiple of both denominators, in this case, 4 and 6. If the total number is not a multiple of both 4 and 6, we would be left with a remainder and would not know whether or not a piece of candy was a certain color. For example, if there are 30 total pieces of candy, there should be 7.5 pieces of orange candy, which is impossible. Thus, 12, the only multiple of 4 and 6.
9. D. The “value of  $\frac{a}{b}$ ” is a ratio, telling us how many apples there are relative to bananas. Assume that there are only 7 pieces of fruit in the bowl, since the denominator of  $\frac{3}{7}$  tells us the total number of fruit is 7 (or a multiple of 7). Since this is the case,  $a$  apples +  $b$  bananas = 7, the total number of pieces of fruit. Since we know that  $a = 3$  (from the probability of picking an apple of  $\frac{3}{7}$ ), we know that  $7 - 3 = b = 4$ . We know that the ratio of apples to bananas, then, is  $\frac{3}{4}$ . There may actually be more than 3 apples and 4 bananas, but they will always stay in this proportion.
10. H. The amount of time a commercial is not airing is  $90 - 20 = 70$  minutes. Therefore, the probability of a commercial not airing is 70 minutes of non-commercial time  $\div$  90 minutes total time, which simplifies to  $\frac{7}{9}$ .
11. B. If there are 36 green bottles, then there are  $60 - 36 = 24$  bottles that are not green. The question asks us what the probability is that a ring will not land on a green bottle; this means that the thing that we “want” to happen is for the ring to not land on a green bottle. Since there are 24 not-green bottles out of a total of 60 bottles, the probability is  $\frac{24}{60} = \frac{2}{5}$ .
12. E. Before Leo loses his pens, he has a total of  $8 + 6 + 2 = 16$  pens. If he loses 4 pens in total, then he is left with only 12 pens. Since we know that 2 of the 4 lost pens are blue, then of the 12 remaining pens, only 4 are blue. Thus, Leo’s chances of picking out a blue pen randomly are 4 out of 12, or  $\frac{1}{3}$ .
13. B. Initially, the bucket of 20 apples contained 8 red apples, since  $20 - 12 = 8$ . If 2 green apples are then removed from the bucket, that leaves the bucket with 18 apples, 8 of which are still red, and 10 of which are green. If one more apple is chosen at random (in this case, “removed from the bucket”), the chances of it being red are 8 out of 18 total apples. This simplifies to 4 out of 9.



14. G. Initially, the bookshelf holds a total of  $12 \text{ fiction} + 15 \text{ nonfiction} = 27$  books. 5 are removed, 1 of which is nonfiction, meaning the other 4 are fiction. The bookshelf now contains  $8 \text{ fiction} + 14 \text{ nonfiction books} = 22$  books. If one more book is removed, the chance of it being fiction is  $\frac{8}{22} = \frac{4}{11}$ .
15. C. Initially, there must have been  $\frac{2}{5} \times 30 = 12$  white pawns in the box, which means  $30 - 12 = 18$  black pawns in the box. If the probability of drawing a white and a black pawn from the box is exactly 50%, after adding in more white pawns, that must mean that the number of white and black pawns are equal, at 18. Therefore,  $18 - 12 = 6$  white pawns were added to the box.
16. G. If no cherries are sold, then the number of cherries must remain at 7 even after other fruit are sold. If the probability of picking a cherry after other fruits are sold must be 50%, then we know that the total remaining number of fruit (including the 7 cherries) must be twice the number of cherries remaining. This means the total number of fruit after some are sold is 14, with 7 being cherry. Of the other 7, we don't know how many remaining are oranges or lemons; we only know that where there were 17 oranges and lemons before, now there are only 7. This means a total of 10 oranges or lemons were sold. Since the question asks what the fewest number of lemons sold was, we can assume that as many oranges as possible were sold. Since this is 8, and 10 were sold in total, that means the minimum number of lemons that could have been sold is 2.
17. A. If 75% of the 20 peppers are yellow, then there are 15 yellow and 5 orange. The probability of both peppers being orange (without replacement) is  $5 \div 20 \times 4 \div 19 = 1/19$ .
18. 0.04. If  $\frac{2}{5}$  of the pears are Bartlett and  $\frac{7}{12}$  are Bosc, then there are 24 Bartletts and 35 Boscs, meaning there is only 1 Anjou. If all the Boscs are removed, then there will be 25 pears remaining, and the probability of picking an Anjou will be 1 out of 25, or 0.04.
19. 0.5. If  $\frac{1}{3}$  of the 48 bottles are apple, then there are 16 apple, and a total of 32 orange and grape. If there are 3 times as many orange as grape, then there must be 24 orange and 8 grape, meaning the probability of picking a bottle of orange is 24 out of 48, or 0.5.
20. 18. If  $\frac{3}{8}$  of the box is milk, then  $\frac{5}{8}$  of the box is dark and white. There is a total of 30 dark and milk, meaning the total number of chocolates is 48, so there must be 18 milk chocolates.
21. 0.2. If Shonda removed 8 movies, 2 of which are dramas, then she removed 6 comedies, leaving 16 dramas and 4 comedies, for a new total of 20 movies. The probability of picking a comedy is now 4 out of 20, or 0.2.
22. 24. If  $\frac{4}{9}$  of Harold's 36 apps are games, then he has 16 games and 20 other apps. The same 20 other apps must eventually represent  $\frac{1}{3}$  of all the apps on his phone, meaning there must be a total of 60 apps, so he needs 24 more games.
23. 0.75. This is a compound probability problem, meaning we set up a multiplication equation:  $\frac{1}{5} \times x = \frac{3}{20}$ . Using either basic algebra or our fact families, we know that  $\frac{3}{20} \div \frac{1}{5} = \frac{3}{4}$ , or 0.75.

## Measures of Central Tendency

1. B. To calculate the mean, we must add up the total number of birds spotted during all the days and then divide by the number of days. To get the total number of birds, multiply each number of birds

by the number of days and add them together.  $1 \times 3 + 2 \times 3 + 3 \times 5 + 4 \times 8 + 5 \times 11 = 3 + 6 + 15 + 32 + 55 = 111$  birds total. To get the number of days, add up the numbers in the second column.  $3 + 3 + 5 + 8 + 11 = 30$  days. Finally, we divide the total birds by the number of days and we get  $111 \div 30 = 3.7$ .

2. G. There were 3 days when 1 bird was spotted, 3 days when 2 birds were spotted, 5 days when 3 birds were spotted, etc. We can list out the number of birds spotted each day as follows: 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5. The list has 30 numbers in it, which is even so there are two middle numbers. Since both middle numbers are 4, the median is 4. Another way to solve this problem without writing out the entire list is to notice that there are a total of 30 days (get this by adding up the second column of the chart). Therefore, the middle numbers will be the 15<sup>th</sup> and 16<sup>th</sup> numbers. These must both be 4 (because there are 11 numbers that are 1's, 2's, and 3's, and 11 numbers that are 5's).
3. A. The range is the highest number of birds spotted minus the lowest number of birds spotted.  $5 - 1 = 4$ .
4. G. The median is the middle number. Cross off numbers from both sides until you reach the middle. There are two middle numbers, which are both 8, so the median is 8.
5. C. The mode is the number that appears the most often. The number 8 appears 6 times, which is more often than any other number, so 8 is the mode.
6. G. To find the mean, add up all the scores and divide by the number of scores. The total of the scores is 158 and the number of scores is 20. Therefore, the mean is  $158 \div 20 = 7.9$ .
7. D. A mean is the sum of all the values in a given set, divided by the total number of the values in that set. In this case, the set consists of 3 numbers:  $x$ ,  $4x$ , and  $5x$ . Adding these together, we arrive at  $10x$ , and must divide by 3 (the total count of numbers in the range) to arrive at the mean, which we are told is 90. So,  $10x \div 3 = 90$ . Solving for  $x$ , we multiply both sides by 3 to arrive at  $10x = 270$ , and then divide both sides by 10 for  $x = 27$ . The question asks for the value of the largest of these, which is  $5x$ , so  $5 \times 27 = 135$ .
8. G. The median is the middle value in a set when all values are in order. If one more student takes the quiz, there will be 21 values, so the 11<sup>th</sup> value will be the median. If one student scores a 10, two students score a 9, and now four students score an 8, that accounts for seven values. The four 7's mean that the median is 7.
9. B. If  $a$  is any number 8 or smaller, then the new median will be 8. If it is 9 or larger, then the new median will be 9. 8 is not an answer choice, so the answer must be 9.
10. F. A mean is a sum of numbers in a range divided by the count of numbers in that range. In this case, the sum is  $6 + x + y$ , which, when divided by 3, equals 10. Simplify to  $6 + x + y = 30$ . Subtracting 6 from both sides, we arrive at  $x + y = 24$ .
11. B. If the total number of books is 345, and Chrissy owns 135, that means 210 remain between Jack and Janet. To find the mean, we divide their sum by 2:  $210 \div 2 = 105$ .
12. H. Since we can find a mean by dividing the sum of the values by the number of values in the set, we can therefore use our fact families to know that multiplying a mean by the number of values in the set results in the sum of the values.
13. C. The problem tells us that Brenda's first three scores add up to 252 and her fourth score is 96. Therefore, the total of all the scores is  $252 + 96 = 348$ . To calculate the mean, we divide the sum by the number of scores, which is 4. We get  $348 \div 4 = 87$ .
14. H. Ellen wants her test mean to be 95. In other words, when she adds up all her scores and divides by 6, she should get 95. Working backwards, this means that if we multiply 95 by 6, we will get the sum of all the scores.  $95 \times 6 = 570$ .

15. A. Let  $x$  represent Brandon's score on the fourth test. Then his mean for the four tests can be represented as  $\frac{3 \times 90 + x}{4}$ . We have to multiply 3 times 90 in the numerator because there were three tests with a mean of 90. We divide by 4 because there were 4 tests total. This expression simplifies to  $\frac{270 + x}{4}$ . Brandon's mean on the first four tests was 85, so we can make an equation  $\frac{270 + x}{4} = 85$ . Multiply both sides by 4. The equation becomes  $270 + x = 340$ . Subtract 270 from both sides. The answer is  $x = 70$ .
16. E. Let  $x$  represent the extra number that is being added to the set. The total of the new set is  $8 \times 20 + x$  because there are 8 numbers with a mean of 20 and we are adding  $x$  to them. This total simplifies to become  $160 + x$ . In order to get the new mean, the total must be divided by 9 because there are 9 numbers including the new one. Therefore, the new mean is  $\frac{160 + x}{9}$ . The problem tells us that the new mean is 4 less than the old mean (which was 20) so the new mean is 16. Therefore, we get  $\frac{160 + x}{9} = 16$ . To solve this equation, first multiply both sides by 9 and then subtract 160 from both sides. The answer is  $x = -16$ .
17. A. No matter how many pets the new student has, the median will remain 1. This means that the new mean should also be 1. We can therefore find the unknown value by setting up an algebraic equation:  $\frac{0 + 0 + 0 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 2 + 3 + x}{13} = 1$ . Combining like terms and cancelling out the denominator gives us  $13 + x = 13$ , so  $x = 0$ .
18. G. If Daniel scored a mean of 10 points in each of 5 games, then the total number of points he scored is  $10 \times 5 = 50$ . We know that he scored at least 1 point in each game. The highest number of points he could score in one game would be 46, because he could score 46 points in one game and then only 1 point in each of the other games.  $46 + 1 + 1 + 1 + 1 = 50$ , so the answer is 46.
19. C. If the average of 3 integers is  $-54$ , then three integers must be  $-55$ ,  $-54$ , and  $-53$ . The least of these is  $-55$  (it is further left on the number line). If we subtract 45 from it, we get  $-55 - 45 = -100$ .
20. F. To get the mean, we must add up the total scores of all the students and divide by the number of students. To get the total scores for all ten students we must multiply 92 by the number of girls, multiply 82 by the number of boys, and add these together.  $92 \times 6 + 82 \times 4 = 552 + 328 = 880$  total points. Finally, we divide the total points by the number of students.  $880 \div 10 = 88$ .
21. C. The numbers in M are 15, 20, 25, 30. The median is the middle number, but in this case we have two middle numbers, which are 20 and 25. Therefore, we have to take the mean of 20 and 25, which is 22.5.
22. H. Let  $x$  represent the mean number of hours that Trang studied per day during the first 5 days. Then  $x + 1$  represents the number of hours per day that she studied during the last 3 days. The total number of hours that she studied in the first 5 days is  $5x$ . The total number of hours that she studied during the last 3 days is  $3(x + 1)$ . Therefore, total number of hours that she studied during all 8 days is  $5x + 3(x + 1)$ . To simplify this expression, use the distributive property. We get  $5x + 3x + 3$ . Then combine like terms to get  $8x + 3$ . We know that she studied for a total of 27 hours, so we get the equation  $8x + 3 = 27$ . To solve this equation, subtract 3 from both sides and then divide both sides by 8. We get  $x = 3$ . This means that the mean number of hours she studied per day during the first 5 days was 3, so she must have studied  $3 + 1 = 4$  hours per day during the last 3 days. Therefore, her total in the last 3 days was  $(3 \text{ days}) \times (4 \text{ hours per day}) = 12$  hours.

23. D. Set X contains three numbers whose mean is 5, which means the sum of the three numbers is 15. Set Y is made from those three numbers being tripled, so their sum is 45. It doesn't matter what the individual numbers are.
24. G. If Markus took 5 tests and has an average of 86, then he scored a total of  $86 \times 5 = 430$  points on all 5 tests. If he wants an average of 87, and he can only take 1 more test, then he will need to have a total of  $87 \times 6 = 522$  points on all 6 tests. This is a difference of  $522 - 430 = 92$  points.
25. B. The range of Constance's data is 14 while the range of June's data is 16. Therefore, Constance's range of times is less than June's range of times.
26. E. June's interquartile range is 10 and Constance's interquartile range is 8. The difference between the two is 2.
27. D. For both Constance and June, 30 is the time for quartile 3. In a Box-and-Whisker, 25% is greater than quartile 3. Therefore, each of them had 25 laps that took longer than 30 seconds, for a combined total of 50.
28. F. The range of this data is 500 and the interquartile range is 200. The difference between the two values is 300.
29. C. 500mph is the minimum and 800mph is the median. In a Box-and-Whisker plot, 50% of the data lies between the minimum and the median.
30. E. In a Box-and-Whisker plot, between the third quartile and the maximum is 25% of the data. Since 900 is the third quartile and 1,000 is the maximum, 13 of Oliver's trials were 900 mph or greater.
31. 5. The range is 5 (because the greatest number of items bought is 5 and the least is 0) and the mode is 0 (because more customers bought 0 items than any other number), so the range is 5 more than the mode.
32. 2. The median is the middle number. There were initially 50 customers, so one more would make 51 customers. The median would be the 26<sup>th</sup> number of items bought, which would be 2.
33. 1.78. Finding the mean requires finding the total number of items bought by everyone. 3 customers bought 5 items each (15 total), 5 customers bought 4 items each (20 total), 8 customers bought 3 items each (24 total), 9 customers bought 2 items each (18 total), 12 customers bought 1 item each (12 total), and 13 customers didn't buy anything. The final sum is 89 items. Dividing by the total number of customers (found by adding up the right column) gives us  $89 \div 50 = 1.78$ .

## Geometry & Measurements

### Area & Perimeter

1. C. If the perimeter is 28 feet, then 28 is equal to the sum of the sides of the rectangle. If the length, or one side, is 4 feet, then there is a second side that is 4 feet long, so the remaining side is equal to  $28 - 8 = 20 \div 2 = 10$ . So the width of this rectangle is 10 feet. Find the area by multiplying the width times the length:  $10 \times 4 = 40$ .
2. G. If the area of a square is 25 inches, then each side is equal to the square root of 25 inches, or 5 inches. The perimeter is the sum of all the sides, or 4 times the length of one side in a square, or 20 in.
3. C. The area of a triangle is given by the formula  $A = \frac{1}{2}bh$ , where  $b$  is the base of the triangle and  $h$  is the height of the triangle. The area is given as 24 square centimeters, so the equation is  $24 = \frac{1}{2}bh$ , which simplifies to  $48 = bh$ . The base is given as 6, so we know that  $h = 8$ .



4. E. If the area of the square is 72 square centimeters, then triangle GLH has an area equal to  $\frac{1}{4}$  of that, or 18 square centimeters. GL and LH are equal, and the area of a triangle is equal to  $\frac{1}{2}bh$ . So,  $18 = \frac{1}{2}(GL)(LH)$  and  $36 = (GL)(LH)$ . Since  $GL = LH$ , both are equal to 6.
5. B. The easiest way to solve this question is to imagine a complete rectangle measuring 7 by 12, with an area of 84. Since a piece is missing, measuring 5 by 6 and having an area of 30, we simply subtract the square from the rectangle.  $84 - 30 = 54$ .
6. E. Bedroom A is 8 by 6.5 1-inch squares. If each 1-inch square is equal to 9 square feet, then each 1-inch square is equal to 1 square yard. Since there are 3 feet per yard, a 3 foot (or, 1 yard) by 3 foot (1 yard) square results in a 9 square foot (1 square yard) area. The area of Bedroom A is simply  $8 \times 6.5 = 52$  square yards.
7. B. If the width of the rectangle is 18 feet, and the length is 9 less than twice that, then the length is  $2(18) - 9 = 27$ . Thus, the total area of the room is 18 ft. wide  $\times$  27 ft. long = 486 sq. ft. Each square porcelain tile measures 3 by 3 feet long, and has an area of 9 sq. ft. Therefore, it will take  $486 \div 9 = 54$  tiles to cover the floor.
8. F. There are several ways to solve this question. We can start by finding the area of each smaller rectangle, which is  $xy$ , and the area of the larger rectangle, which is therefore  $3xy$ . If one is tiling a rectangular area  $8y$  by  $9x$ , then the total area to be tiled is  $72xy$ . Divide to find how many sets of area  $3xy$  will be needed to tiled an area of  $72xy$ , which is 24.
9. C. The total area of the kitchen is 16 ft.  $\times$  12 ft. = 192 sq. ft. We can tell that each square marble tile will measure 4 ft. by 4 ft., since we can divide the total length and width of the kitchen by the number of tiles that fit into that measurement. Each square marble tile therefore has an area of  $4 \times 4 = 16$  sq. ft. The top two partially-shaded areas are  $\frac{2}{3}$  of a full tile, or  $\frac{32}{3}$  sq. ft. each. Both top shaded areas represent  $\frac{64}{3}$  sq. ft. The bottom two partially-shaded areas are about half of the size of the top two partially-shaded areas, or  $\frac{32}{3}$  sq. ft. Together, they represent  $\frac{64}{3} + \frac{32}{3} = 32$  sq. ft. Subtracting that from the total area leaves  $192 - 32 = 160$  sq. ft.
10. G. Draw these triangles to help visualize. They share a base, so the two shapes form a lopsided diamond. The shorter triangle has a height of  $x$ , and the larger triangle has a height of  $2x$ . We know that the area of one triangle is  $A = \frac{1}{2}bh$ . For the small triangle, this is  $A = \frac{1}{2}5x$ , and for the larger triangle, this is  $A = \frac{1}{2}5(2x)$ . The sum of these is 60, so we write:  $60 = (\frac{1}{2})(5x) + (\frac{1}{2})(10x)$ . Multiply all terms by 2 for  $120 = 5x + 10x$ , and simplify for  $120 = 15x$ , and  $x = 8$ . The question asks the height of the larger, which is  $8 \times 2 = 16$ .
11. A. If the area of the square is 16, then each of its sides is  $\sqrt{16} = 4$ . This means that the side shared by the square and the triangle (the dotted line) has length 4. If the perimeter of the triangle is 12, then we subtract 4 from the total perimeter to find the length of the remaining 2 sides of the triangle, which must be 8. The other 3 sides of the square have a total length of  $3 \times 4 = 12$ , so the total perimeter must be  $12 + 8 = 20$ .
12. F. Since each side of square ACGE has is 4 cm, it has an area of 16 cm. The ACGE square has been split into 4 smaller squares, each with an area of 4. The small triangles in each small square each

have an area of 2, since they are half the size of each small square. Triangle CJG is congruent to triangle CHG, so the area of the two triangles together is 8.

13. A. A hexagon has 6 sides. The perimeter of a polygon is the sum of the lengths of its sides. In this case, the lengths of a hexagon are given. The total perimeter can be represented as  $54 = 2(x) + 2(3x) + 4x + 6$ . Simplify and solve for  $x$ :  $48 = 12x$ , and  $x = 4$ .
14. H. A regular polygon has equilateral sides, meaning all sides have the same length. The hexagon, then, has 6 sides that are each 8 meters long, meaning the total perimeter is  $8 \times 6 = 48$  meters. If a square has the same perimeter, but only has 4 sides, then the length of each side is  $48 \div 4 = 12$  meters. The area of the square is  $12^2 = 144$  square meters.
15. B. Draw this shape. A regular pentagon has 5 sides, so each side must be  $15 \div 5 = 3$  mm long. If this is the base of the triangle, and the hypotenuse is 5 mm, then we know that the height of the triangle is 4 mm because this is a special right triangle (a 3-4-5 triangle). This means the area is  $0.5(3)(4) = 6 \text{ mm}^2$ .
16. F. We must find the lengths of PQ, QS, ST, and PT. QS is given as 5. We know that PQ = 3 because Q bisects PR, meaning PQ = QR = 3, and that PR has a total length 6. ST = RS because S bisects RT. We know that RS = 4 because it is a special right triangle (a 3-4-5 triangle). Since  $b = RS = ST = 4$ , then length RT = 8. Since PR = 6 and RT = 8, and the triangle is a right triangle, we know that PT = 10 (because it is proportional to QRS and also because it is a multiple of the 3-4-5 triangle: a 6-8-10 triangle). Therefore, PQ = 3, QS = 5, ST = 4, and PT = 10. The sum is the perimeter:  $3 + 5 + 4 + 10 = 22$ .
17. D. The area of a circle can be determined using the formula  $A = \pi r^2$ , where  $r$  is the radius of the circle. The radius is given as 10, so the area of the circle is  $100\pi$ , or approximately 314 sq. ft. Since a gallon of paint can cover 30 sq. ft.,  $314 \div 30 = 10.46$ . We need to round up to the nearest whole gallon; 10 gallons of paint will only cover 300 sq. ft. In order to cover the remaining 14 sq. ft., an 11<sup>th</sup> gallon would need to be purchased.
18. F. We are given circumference and told to find area. A circumference of  $20\pi$  means the diameter is 20, so the radius is 10, and the area is therefore  $100\pi$ . However, the problem asks for the area of a single slice, which is  $1/8$  of the pie.  $100\pi \div 8 = 12.5\pi$ .
19. B. The circumference of a circle is equal to  $2\pi r$ , or  $\pi d$ . In the figure, KJ is the diameter of the circle. Since JKLM is a square, to find the length of KJ, divide the perimeter 16 by the 4 sides. This tells us that  $KJ = \pi(4) = 4\pi$ .
20. G. Since we know the area of the circle, we can find the radius, which we know will be the measure of AB.  $36\pi = \pi r^2$ . Solving for  $r$ , we can cancel out the  $\pi$  and take the square root of 36, to find that  $r = 6$ . Now we know that AB = 6. Since AC = 2AB, then AC =  $2(6) = 12$ . Using the formula for the area of a triangle,  $\frac{1}{2}(b)(h)$ , we can plug in values for both  $b$  and  $h$ :  $\frac{1}{2}(6)(12) = 36$ .
21. C. Since the area of the circle is  $36\pi$ , we can solve for the radius of the circle:  $36\pi = \pi r^2$ . This means that the radius of the pie is 6. Since Amy is looking for the smallest possible square box that will fit the pie, we know that the side of one length of the box should be equal to the diameter of the pie, which is  $6 \times 2 = 12$ . The question asks for the perimeter of the box, which is  $12 \times 4 = 48$  inches.
22. H. Since the area of the circle is  $64\pi$ , we can solve for the radius of the circle:  $64\pi = \pi r^2$ . Dividing both sides by  $\pi$ , we arrive at  $64 = r^2$ . Squaring both sides, we find that  $r = 8$ . Since the circle is inscribed in the circle, its center is exactly in the middle of the square. The length of a square is equal to the diameter of the circle, or  $2r = 16$ . The area of the square is equal to  $A = l \times w$ , or  $16 \times 16 = 256$ .
23. B. We can determine the total surface area of a cylinder by using the formula  $A = 2\pi rh + 2\pi r^2$ , which represents the area of the cylinder including its top and bottom bases. We are given  $h = 10$  in., and  $d$

= 4 in. We can convert  $d$  diameter into  $r$  radius by halving the value of  $d$ , so  $r = 2$  in. Substitute the values into the formula:  $A = 2\pi(2)(10) + 2\pi(2)^2$ . This simplifies to  $A = 40\pi + 8\pi$ , or  $48\pi$ .

24. G. The area of a trapezoid can be determined using the formula  $A = \frac{a+b}{2}h$ , where  $a$  and  $b$  are the lengths of the top and bottom bases (the parallel sides) and  $h$  is the height of the trapezoid (the distance between  $a$  and  $b$ ). Substituting the values from the figure:  $A = \frac{10+20}{2}(10)$ , which simplifies to 150.
25. B. Since AB is 12 cm. long, so too is DC. This is because the opposite sides of a parallelogram have the same length. Since E is located at the midpoint of DC, we know DE and CE are both 6. If we can find the height of AE, then we can determine the area of the rectangle. Since  $24 = \frac{1}{2}(b)(h)$ , where  $b =$  DE and  $h =$  AE, then  $\frac{1}{2}(6)(h)$ . Solving for  $h$ , we find that AE = 8. The area of a parallelogram is simply  $A = bh$ . In this case,  $12 \times 8 = 96$ .
26. 30. If the length is one more than the width, we can represent this as  $l = w + 1$ . The perimeter would be  $2(w) + 2(w + 1) = 22$ , or  $4w + 2 = 22$ . To find  $w$ , subtract 2 from both sides then divide by 4 for  $w = 5$ . The width is 5, which means the length is  $22 = 5 + 5 + 2l$ , or  $12 = 2l$ , for  $l = 6$ . The area is  $l \times w$ , or  $5 \times 6 = 30 \text{ in}^2$ .
27. 100. The perimeter of the pentagon is 40 inches, since it is regular and has 5 equally long sides ( $5 \times 8$ ). If a square has the same perimeter as the pentagon, its side length is  $40 \div 4 = 10$  inches, and its area is 100 square inches.
28. 4. If the side lengths must all be integers, then the rectangle could only have side lengths of 1 and 24, 2 and 12, 3 and 8, or 4 and 6, which is 4 different combinations. All four of these possibilities render different perimeters, of 50, 28, 22, and 20, respectively.
29. 48. Draw this shape out so that the circle is inside the square (refer to #20 for an example). We know that the circle has a radius of 6, since the area is  $36\pi = \pi r^2$ . This is half the length of one side of the square, so one side of the square is 12 in long. This means the perimeter is  $12 \times 4 = 48$  in.
30. 50. Draw this shape out so that the square is inside the circle, with the corners touching the circle. We know that the circle has a radius of 5, since the area is  $25\pi = \pi r^2$ . This means the distance from the center of the square to each corner is 5 mm. There are many ways to find the area of the square. We can solve this without knowing the formula for the Pythagorean Theorem. Draw a radius from the center of the circle to the top-right corner of the square, and do the same to the bottom-right corner of the square. Both have length 5, so we know the area of the triangle formed is  $0.5(5)(5)$ , or  $12.5 \text{ mm}^2$ . There are 4 of these triangles in the square, so  $12.5 \times 4 = 50$ .
31. 4. Two circles that share the same center are called concentric. Draw one circle, and label the radius  $x$ . Then, surrounding it, draw a larger circle, and draw a new radius, labeling the radius  $2x$ . The area of the small circle is  $\pi x^2$ . The area of the larger circle is  $\pi(2x)^2$ , or  $\pi 4x^2$ . Choose an easy number to work with (use a small integer that isn't 0 or 1, when using this method), like 2. Substitute this in for  $x$ . This means the small circle would have an area of  $4\pi$ , while the area of the large circle would be  $16\pi$ . The larger circle has an area that is 4 times as large as the area of the smaller circle.
32. 42. The bases of this parallelogram are 6 units long, and the height from one base to the other is 7 units. The area of a parallelogram is base times height, or  $6 \times 7 = 42$ .

## Volume

1. B. The volume of a rectangular prism like the swimming pool can be determined by the formula  $\text{length} \times \text{width} \times \text{height}$ . We are already given the  $\text{length} \times \text{width}$ , which is 350 sq. meters. We need only solve for the height:  $4,200 = 350 \times \text{height}$ . By dividing both sides of the equation by 350, we arrive at a height/depth of 12 meters.
2. F. The volume of a rectangular prism like the swimming pool can be determined by the formula  $\text{length} \times \text{width} \times \text{height}$ . We are given all three dimensions, but must be careful to convert units to feet. The volume, then, is:  $20 \text{ ft.} \times 40 \text{ ft.} \times 0.5 \text{ ft.} = 400 \text{ cu. ft.}$
3. B. First, determine the volume of each package. The volume of a cube measuring 2 feet on all sides is  $2^3 = 8$  cubic feet. Next, determine the volume of the storage area:  $16 \times 24 \times 4 = 1,536$  (note that the height allows for two layers of packages).  $1,536 \text{ cubic feet} \div 8 \text{ cubic feet per package} = 192$  packages.
4. E. The cube has a volume of  $3^3 = 27$  cubic inches. The pyramid has a volume of  $\frac{1}{3}(3)(3)(1) = 3$ . In total,  $27 + 3 = 30 \text{ in}^3$ .
5. A. The volume of a sphere is given by the formula  $V = \frac{4}{3} \pi r^3$ . If the volume is  $36\pi$ , first multiply both sides by 3 to simplify, for  $108\pi = 4\pi r^3$ . Then, divide both sides by  $4\pi$  for  $27 = r^3$ . This is the perfect cube of 3, or  $3 \times 3 \times 3$ .
6. F. If the area is  $288\pi$ , first multiply both sides by 3 to simplify, for  $864 = 4\pi r^3$ . Then, divide both sides by  $4\pi$  for  $216 = r^3$ . This is the perfect cube of 6, or  $6 \times 6 \times 6$ . The circumference is  $2\pi r$ , or  $12\pi$ .
7. D. The volume of any pyramid can be found by using the formula  $\frac{1}{3}bh$ , where  $b$  = area of the triangle's base, and  $h$  = the height of the pyramid. Here, we are given that  $128 = \frac{1}{3}b(6)$ .  
Simplifying, we have  $64 = b$ . Since the base is a square, we can take the square root of both sides to find that each side is 8 ft. long.
8. G. The volume of any pyramid can be found using the formula  $\frac{1}{3}bh$ , where  $b$  = area of the triangle's base, and  $h$  = the height of the pyramid. Substituting in known values, we arrive at  $36 = \frac{1}{3}(12)h$ . Simplify and solve:  $36 = 4h$ , so  $h = 9$ .
9. D. Using the formula of the volume of a pyramid  $\frac{1}{3}bh$ , we can substitute known values:  $144 = \frac{1}{3}b(9)$ .  
Isolate  $b$ :  
 $48 = b$ . The question does not ask for a particular side, only what the possible sides could be. In this case, we know that the area must be  $48 \text{ in}^2$ . The only combination of dimensions provided that does this is 6 in. by 8 in. ( $6 \times 8 = 48$ ).
10. E. Since we are already given the value of the  $\text{length} \times \text{width}$  as being  $24 \text{ in}^2$ , we solve for the height using the volume:  
 $84 = 24 \times h$ . Simplifying, we find that  $h = 3.5 \text{ in.}$
11. B. The volume of a trapezoidal prism can be determined using the formula  $\frac{a+b}{2} \times h \times l$ , where  $a$  and  $b$  represent the widths of the trapezoid's bases,  $h$  represents the height of the prism, and  $l$  represents the length of the prism. We are given all 4 values, and need only to substitute:  $\frac{4+8}{2} \times 5 \times 10 = 300$ .



12. G. The volume of a rectangular prism can be determined by multiplying the length by the width by the height. In the case of the top prism, the volume can be expressed as  $y \times y \times y = y^3$ . In the case of the bottom prism, we know that the height  $= y$ . The length and the width are shown as 2 more than the length and width of the top prism, which we can represent as  $y + 2$  and  $y + 2$ . Thus, the bottom prism has a volume of  $y(y + 2)(y + 2)$ . We add this to  $y^3$  to arrive at  $y^3 + y(y + 2)(y + 2)$ .
13. D. The beachball has radius of 3, since  $9\pi = \pi r^2$ . This means the volume is  $\frac{4}{3}\pi(3)^3$ , or  $36\pi$  inches<sup>3</sup>.
14. 960. The box can fit 12 cubes across, 10 cubes deep, and 8 cubes high, so the total number of cubes is  $12 \times 10 \times 8 = 960$ .
15. 3. The volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ , or one-third the height of the cone times the area of the cone's base. If the volume is  $16\pi$  and the radius of the base is 4, then  $16\pi = \frac{1}{3}(4^2)(\pi)(h)$ . This simplifies to  $48\pi = 16\pi h$ , and  $h = 3$ .
16. 8. The volume of a cylinder can be found by  $V = \pi r^2 h$ . If the volume is  $48\pi$  and the radius is 2, then we have  $48\pi = (\pi)(4)(h)$ . This leaves us with  $h = 12$ . If the container is  $\frac{2}{3}$  full, then the depth must be 8 inches.
17. 90. If the pool has a depth of 3 feet, then the volume of water in it is  $12 \times 50 \times 3 = 1,800$ . At a rate of 20 cubic feet per minute, it will take  $1,800 \div 20 = 90$  minutes.
18. 4. The volume of a pyramid can be found by  $V = \frac{1}{3}lwh$ . If  $V = 32$  and  $h = 6$ , then  $32 = \frac{6lw}{3}$ . Solving for  $lw$ , which is the area of the base, we arrive at  $96 = 6lw$  and  $lw = 16$ . Since the base is a square, we know that  $l = w = 4$ .
19. 6. If the volume of a filled beachball is  $36\pi$ , then its radius can be found by solving for  $r$  in  $36\pi = \frac{4}{3}\pi r^3$ . Multiply both sides by 3 for  $108\pi = 4\pi r^3$ . Divide both sides by  $4\pi$  for  $27 = r^3$ , and  $r = 3$ . The circumference is  $2\pi r$ , or  $6\pi$ . The volume is 6 times bigger.

## Triangles

1. B. The longest side of a right triangle is the hypotenuse, or the side that is opposite the  $90^\circ$  angle. In this figure, there are three right triangles: WXY, WYZ, and WXZ. The largest of these triangles is WXZ, which encompasses both of the other right triangles. Therefore, the largest hypotenuse must be opposite the  $90^\circ$  angle XWZ, or length XZ.
2. G. The sum of the interior angles of a triangle always equals  $180^\circ$ . This means that  $x + y + z = 180$ . Since the question asks for the value of  $x + y$ , we simply subtract  $z$  from both sides to arrive at  $x + y = 180 - z$ .
3. D. In degrees, the sum of the interior angles of a triangle always equals  $180^\circ$ . Thus, we know that  $y^\circ = 180^\circ - 42^\circ - 76^\circ$ . So,  $y^\circ = 62^\circ$ . A straight line has a degree measurement of  $180^\circ$ ; 2 angles that add up to  $180^\circ$  are called supplementary angles. So,  $w^\circ = 180^\circ - 76^\circ$ , or  $104^\circ$ . Similarly,  $x^\circ = 180^\circ - 42^\circ$ , or  $138^\circ$ . Thus,  $62^\circ + 104^\circ + 138^\circ = 304^\circ$ .
4. H. Since the sum of interior angles of a triangle is always equal to  $180^\circ$ , then  $t + u + v = x + y + z = 180^\circ$ . We don't know for sure that any of the other statements are true, since the degree measure individually could vary wildly from one triangle to another.
5. C. Since the sum of interior angles of a triangle is always equal to  $180^\circ$ , and we have the measurement of two of the three angles, we can find the third:  $180 - 90 - 40 = 50$ . Note that the

question asks for the value of  $y$ . Note that the  $50^\circ$  angle and angle  $y$  are supplementary (they add up to  $180^\circ$ ) because the line formed by the base of the triangle extends beyond the triangle itself. The degree measurement of a straight line is  $180^\circ$ . Since the two angles are supplementary,  $y = 180 - 50$ , or  $130^\circ$ .

6. F. Angle RSQ is supplementary to the exterior angle labeled as  $125^\circ$ . Therefore, angle RSQ =  $180 - 125 = 55^\circ$ . Knowing this and the fact that angle SQR =  $20^\circ$ ,  $x = 180 - 55 - 20$ , or  $105^\circ$ .
7. B. All angles in an equilateral triangle are congruent, so each angle in triangle ABC is equal to  $60^\circ$ . Triangle BCD is a 30-60-90 right triangle, since we know that BD bisects and is perpendicular to AC. Since BDC =  $90^\circ$ , and BCD =  $60^\circ$ , DBC =  $30^\circ$  (since the sum of interior angles of all triangles must equal  $180^\circ$ ).
8. F. We can see that the bigger triangle can in fact be split into two smaller triangles. The triangle on the right has measures  $90^\circ$ ,  $50^\circ$ , and  $2y^\circ$ . Since the sum of all interior angles in a triangle must equal  $180^\circ$ , then  $180 = 90 + 50 + 2y$ . Solve for  $y$  by simplifying to  $40 = 2y$  and  $y = 20$ . In the triangle on the left, the angle measures are  $90^\circ + 3(20)^\circ + x$ , which must also equal 180. Simplify and solve for  $x$ :  $180 = 90 + 60 + x$  becomes  $x = 30$ .
9. B. If  $x = 35$ , then the value of angles RQS and QRS =  $35 + 70 = 105$ . Since we know this, then angle QSR =  $180^\circ - 105^\circ = 75^\circ$ , since QSR is the vertical angle to angle TSU, which means it too is  $75^\circ$ , then TSU ( $y^\circ$ ) =  $75^\circ$ .
10. F. Since AC is a diagonal that divides the rectangle in half, the area of ACD is  $\frac{1}{2}$  (15), or 7.5 cm.

Applying the formula for area of a triangle, we know that  $A = \frac{1}{2}bh$ . The height in this case is 3 cm, so  $7.5 = \frac{1}{2}b(3)$ , or  $7.5 = 1.5b$ . If we solve for  $b$ , we find that  $b = 5$ .

11. C. The fact that  $PQ = QR = PR$  tells us that the big triangle is an equilateral triangle, meaning its three angles ( $\angle QRP$ ,  $\angle PQR$ , and  $\angle RPQ$ ) each measure  $60^\circ$ . This is a special property of all equilateral triangles regardless of how long their sides are. The fact that  $ST = SU$  and that the measure of  $\angle TSU = 90^\circ$  tells us that the small triangle is another special type of triangle, where the remaining angle measures are equal, since the lengths of the other sides are equal. Thus, the other two remaining angles are each  $45^\circ$ . Since  $\angle RST = 60^\circ$ , and  $\angle QRP = 60^\circ$ , then so too must  $\angle RTS$ , since the sum of the angles in triangle RST must equal  $180^\circ$ . If we know this, and we know that  $\angle STU = 45^\circ$ , then  $\angle UTQ$  must equal  $180^\circ$  (the degree measure of the straight line RQ) minus  $60^\circ$  ( $\angle RTS$ ) minus  $\angle STU = 45^\circ$ . Thus,  $\angle UTQ = 75^\circ$ .
12. G. Since QR and TU are parallel, the angles QSR and TSU are vertical and share the same degree measurement. Since both QSR and TSU are triangles, the sum of interior angles in each is  $180^\circ$ . This means that  $x + y = 35^\circ + 45^\circ = 80^\circ$ .
13. B. AKM is a right triangle comprised of 4 smaller triangles. For one shape to be congruent to another, both shapes must have exactly the same shape and size, including the lengths of all sides and the degree measurements of all angles. There are 7 triangles congruent with AKM: ACM, CMO, CEO, AKC, KCM, MCE, and MEO.
14. 104. Any exterior angle of a triangle is equal to the sum of the other two angles inside the triangle. So we write  $x + 3 + 3x + 5 = 7x - 43$ , which simplifies to  $4x + 8 = 7x - 43$ . Solving for  $x$  results in  $x = 17$ . Exterior angle DBC is therefore  $7(17) - 43 = 76$ , so the measure of interior angle ABC is  $180 - 76 = 104$ .

15. 12. The triangles are both 3-4-5 right triangles. The larger triangle has sides three times the size of the smaller triangle, since  $5 \times 3 = 15$ , so its dimensions are 9-12-15.
16. 4. Angles CDE and CAB are alternate interior angles, so they are congruent. Therefore,  $8x + 7 = 39$ , meaning  $8x = 32$  and  $x = 4$ .
17. 12. The area of a triangle is  $\frac{1}{2}bh$ , or  $84 = \frac{1}{2}(14)(h)$ , which simplifies to  $84 = 7(h)$ . So, the height must be  $84 \div 7 = 12$ .
18. 25. Looking at right triangle ABD, we see that the hypotenuse is 20 and one of the legs is 12. We can use the 3-4-5 right triangle ratio to figure out that side AD is 16. Looking at right triangle BDC, we see that the hypotenuse is 15 and one of the legs is 12. We can use the 3-4-5 right triangle ratio to figure out that side DC is 9. Therefore,  $AC = 16 + 9 = 25$ .
19.  $-120$ . There are  $180^\circ$  in every triangle. An equilateral triangle is special in that all angles are also equal. If one angle is  $x$ , then we know that  $3x = 180^\circ$ , so  $x = 60^\circ$ . 180 less than this is  $60 - 180 = -120$ .
20. 76. There are  $180^\circ$  in every triangle. In an isosceles triangle, we know that two of the angles will be equal, and the other will not be equal. Since we know the smallest is 28, then the other two must each be larger than this, as well as equal to each other. This means the sum of the other two angles is  $180 - 28 = 152$ .  $152 \div 2 = 76$ .

## Circles

1. B. A circle has a total of  $360^\circ$  of arc. In  $\frac{5}{6}$  of a circle, then, there are  $360^\circ \div 6 = 60^\circ \times 5 = 300^\circ$ .  $300^\circ - 120^\circ = 180^\circ$ .
2. F. The area of a circle is determined by the equation  $\pi r^2$ , where  $r$  is the length of the circle's radius. Since the area is given, we need to determine the radius in order to determine the diameter. Set the area equal to the formula:  $4\pi = \pi r^2$ . Solve for  $r$  by dividing both sides by  $\pi$ :  $4 = r^2$ . Take the square root of both sides:  $2 = r$ . Since the question asks for the diameter, and the diameter of a circle is twice the radius,  $2(2) = 4$ .
3. D. If the radius ( $r$ ) is 4, then the diameter ( $d$ ) is 8, since  $d = 2r$ . Since circumference is  $C = 2\pi r$ , the circumference is  $8\pi$ .
4. F. Let  $x$  represent the diameter of circle R. If the radius of circle Q is 4 times the diameter of circle R, then in terms of  $x$ , the radius of circle Q can be expressed as  $4x$ . The diameter of circle Q is  $2 \times 4x = 8x$ . Thus, the ratio is 8:1.
5. C. A tire is a circle that rotates along its circumference. The circumference of a circle is  $C = 2\pi r$ . In this case, we know that the diameter is 4 feet, so the radius is 2 feet. Thus,  $C = 2\pi(2)$ , or  $4\pi$ . This simplifies to approximately  $4(3.14) = 12.5$ .
6. F. The circumference of each wheel dictates the distance that must be traveled for the wheel to complete one revolution. This formula is  $2\pi r$ , where  $r$  is the radius of the circle. Let  $x$  represent the radius of the rear wheel. The circumference of the rear wheel is  $2\pi x$ . In terms of  $x$ , the radius of the front wheel is  $3x$ . So the circumference of the front wheel is  $2\pi(3x)$ , or  $6\pi x$ . The circumference of the front wheel is 3 times greater than that of the rear wheel. This means that in 1 revolution of the front wheel, the rear wheel will spin 3 times.
7. C. If the length of each side is 2, then the radius of the half circle is 1. The area of a circle is represented by  $\pi r^2$ , where  $r$  = radius. Since the radius = 1, the area of the full circle would be  $\pi$ . This is only a half circle; divide the area by 2, leaving us with  $\frac{\pi}{2}$ .

8. G. The circumference of a circle is  $C = 2\pi r$ . We can use the circumference of circle A to determine its radius.  $8\pi = 2\pi r$ , so  $r = 4$ . If the radius of circle B is twice that of circle A, then the radius of circle B is 8. The area of a circle is  $A = \pi r^2$ . For circle B, this means  $A = \pi(8)^2$ , or  $A = 64\pi$ .
9. B. Since we have a circle, and since O is the center of the circle, we know that SR is the diameter of the circle, and that OR, OS, and OQ are all radii of the circle, having equal lengths. And since QS = QR, we know that the angle formed by QOR =  $90^\circ$ . Since the sum of interior angles of a triangle (for example, triangle QRO) must equal  $180^\circ$ , the other two angles (of which one is  $x$ ) must equal  $180 - 90 = 90$ . Since OR = OQ because they are both radii, both angles must be equal, and triangle QOR must be an isosceles triangle. Thus,  $2x = 90$  means that  $x = 45$ .
10. H. The diameter of a circle is equal to the 2 times the radius of the circle. Since the radius is represented by  $\overline{AO}$ ,  $2 \overline{AO}$  = the diameter of the circle, which we know to be both  $\overline{EC}$  and  $\overline{BD}$ . Thus,  $\overline{EC}$  and  $\overline{BD}$  are both equal to 18, not 9 or 16. The chords  $\overline{ED}$  and  $\overline{BC}$  formed by the two diameters must have equal lengths, so  $\overline{ED} \neq 9$  while  $\overline{BC} = 7$ .  
**NOTE: There is a **TYPO** in the 2024 edition of the Blue Book that mistakenly lists H as  $2\overline{AO} = \overline{EC} + \overline{BD}$ . This should be corrected to  $2\overline{AO} = \overline{EC} = \overline{BD}$ . This will be corrected in the 2025 edition of the text.**
11. D. The radius of the largest circle can be quantified as the radius of circle R + the diameter of circle Q. Since we know the radius of circle R = 3, we need to add to this  $2 \times$  the radius of circle Q. This is  $3 + 5 + 5 = 13$ . This is the radius of the largest circle. The question asks for the diameter, which is simply  $13 \times 2 = 26$ , since the diameter of a circle is 2 times its radius. This is valid because R is the center of both circle R and the largest circle, and Q is perfectly tangent to circle R and the largest circle (meaning it perfectly overlaps exactly at one point).
12. E. The area of a circle can be determined by the formula  $\pi r^2$ , where  $r$  = radius. Substitute the values of each circle's radius. The area of circle X is  $\pi \left(\frac{1}{3}\right)^2 = \pi \frac{1}{9} = \frac{\pi}{9}$ . The area of circle Y is  $\pi$ . The ratio of the area of circle X to that of circle Y is 1:9, since  $\frac{\pi}{9} \div \pi = \frac{1}{9}$ .
13. C. The area of a square is the length of the side squared, or in this case,  $4^2 = 16$ . The area of the circle is given by the formula  $\pi r^2$ . In this case, the radius is half the length of a side of the square, or 2. Thus, the area of the circle is  $4\pi$ . Subtract the area of the circle from the area of the square:  $16 - 4\pi$ . Note that 4 can be factored out of this expression, but cannot be cancelled out completely. Thus, the expression  $4(4 - \pi)$  would be acceptable, but  $4 - \pi$  would not.
14. 25. The area of a circle is  $A = \pi r^2$ . If the area is  $16\pi$ , then the radius can be found by solving for  $r$ .  $16\pi = \pi r^2$ . Divide by  $\pi$  on both sides, leaving  $16 = r^2$ . This gives us  $r = 4$ . The circumference of a circle is  $2\pi r$ , or in this case,  $8\pi$ .  $\pi \approx 3.14$ , and  $8\pi$  gives us 25.12, which rounds down to 25.
15. 3.14.  $\frac{1}{4}$  of the circle centered at vertex D will be inside the square. The other  $\frac{3}{4}$  will be outside the square. Therefore, since the circle has radius 2 and area  $4\pi$ , the section of the circle inside the square will have an area of  $\pi$ , which is 3.14 when rounded to the hundredths place.
16. 120. Since the radius is 6, and the circumference of a circle is given as  $C = 2\pi r$ , the circumference of this circle is  $(2)(6)(\pi) = 12\pi$ . If arc  $AB = 4\pi$ , then it represents  $\frac{1}{3}$  of the circumference. There are  $360^\circ$  in a circle, so  $\frac{1}{3}$  of the degrees in a circle is  $360 \div 3 = 120^\circ$ .
17. 14. If the area of circle O is  $81\pi$ , then its radius is 9, and the radius of circle P is 7. Therefore, the diameter of circle P is 14.



18. 4. Since the outer circle has a radius of 6, its area is  $36\pi$ . The area of the shaded region is  $20\pi$ , so the inner circle must have an area of  $16\pi$ . Therefore, its radius must be 4.

## Angles

1. B. A  $90^\circ$  angle can only be formed by perpendicular lines, in which case the line formed by drawing through G only intersects line  $l$  once.
2. G. There are  $180^\circ$  of measure in a straight line like MO. If  $45^\circ$  are taken up by angle LNO, then  $180 - 45 = 135^\circ$  remain.
3. D. A straight line, as is represented by  $\overline{XW}$ , has a degree measurement of  $180^\circ$  in total. If  $30^\circ$  is already the measure of  $\angle YQW$ , then the remaining degrees in  $\angle XQY$  must be equal to  $180^\circ - 30^\circ = 150^\circ$ . We are told that  $\angle XQY$  is bisected (cut in half) by  $\overline{QZ}$ , forming two smaller, equally sized angles. To find the measure, we divide the total measure of  $\angle XQY$  in half:  $150^\circ \div 2 = 75^\circ$ .
4. H. Since QR is a line, the sum of all angles on the right and left side each equal  $180^\circ$  (since the total degree measure around the line itself equals  $360^\circ$ ). On the left side, this means that  $5y^\circ = 180^\circ$ , which simplifies to  $y = 36^\circ$ . We can substitute this in on the right side for all instances of  $y$ :  $180^\circ = x^\circ + 3(36^\circ)$ , which simplifies to  $180^\circ = x^\circ + 108^\circ$ . Solving for  $x$ , we find that it equals  $72^\circ$ .
5. C. There are  $360^\circ$  in a circle. Though no circle is shown, we know that the measure of angles around a single point is  $360^\circ$ , formed by an imaginary circle. The degree measures are given individually, but we know that together they form a circle. Thus,  $x + 2x + 3x = 6x$  and  $360^\circ = 6x$ . Solving for  $x$ , we arrive at  $360^\circ \div 6 = 60^\circ$ .
6. G. The measure of angles around a single point (the intersection of the line segments) consists of  $360^\circ$  of arc. This means the only unlabeled angle has a degree measure of  $360^\circ - 135^\circ - 65^\circ - 85^\circ = 75^\circ$ . The arc  $y$  spans this angle as well as the angle given as  $65^\circ$ . Therefore,  $65^\circ + 75^\circ = 140^\circ$ .
7. B. Make use of vertical and supplementary angles to determine the interior angles of the triangle formed by the intersection of the three lines. The left-most interior angle of the triangle measures  $45^\circ$  because it is the vertical (opposite) angle from the angle given as  $45^\circ$ . The right-most interior angle of the triangle measures  $50^\circ$  because it is supplementary to the angle given as  $130^\circ$ , meaning that the sum of both angles must add up to  $180^\circ$  (because the degree measure of a straight line is  $180^\circ$ ). This means that the bottom-most interior angle of the triangle  $= 180^\circ - 45^\circ - 50^\circ$ , which equals  $85^\circ$ , since the sum of all interior angles of a triangle must equal  $180^\circ$ . The bottom-most interior angle of the triangle is supplementary to the angle denoted as  $y^\circ$ , so  $y = 180 - 85 = 95$ .
8. F. There are several ways to solve this question. One of the most efficient is to realize that because there are two parallel lines, all angles formed at the intersection of Q and S correspond (are equal to) the respective angles formed at the intersection of R and S. This means that  $x^\circ = 65^\circ$ , etc. Since the sum of all angles surrounding the intersection of R and S must equal  $360^\circ$ , we need only find the value of the angle not labeled. Since  $x^\circ = 65^\circ$ , and since the angle not labeled is supplementary to  $x$ , the value of the angle not labeled is  $180^\circ - 65^\circ = 115^\circ$ . Thus, the sum of  $x + y + z = 360^\circ - 115^\circ$ , which equals  $245^\circ$ .
9. D. Around the intersection of lines  $r$  and  $q$ , the degree measure equals  $360^\circ$ . At this point, the angle opposite the one given as  $125^\circ$  also equals  $125^\circ$ , since the two angles are vertical angles. The remaining two angles must then have a measure  $55^\circ$  each, since  $(2)(55^\circ) + 2(125^\circ) = 360^\circ$ . This tells us that the left-most interior angle in the triangle formed by lines  $q$ ,  $r$ , and  $s$  is  $55^\circ$ . Since the sum of all interior angles of a triangle must equal  $180^\circ$ , then the right-two interior angles have a sum of  $180^\circ - 55^\circ = 125^\circ$ . The sum of all angles formed to the left of line  $s$  must equal  $360^\circ$ , since there are 4 sets of angles formed at the intersection of  $s$  and  $q$  and  $s$  and  $r$ . Since this is the case, and since we know

that 2 of these 4 angles already have a combined degree measure of  $125^\circ$ , then the other 2 angles (given as  $x^\circ$  and the vertical angle of  $y^\circ$ ) must have a combined degree measure of  $360^\circ - 125^\circ = 235^\circ$ . This means that  $x^\circ + y^\circ = 235^\circ$ .

10. H. When two lines intersect, angles that are opposite from each other are called vertical angles, and always have the same degree measurement. In this case, if  $x = 40^\circ$ , then the vertical angle opposite it is also equal to  $40^\circ$ . The other two angles in the triangle that is formed must then equal  $180^\circ - 40^\circ = 140^\circ$ . Since the two angles themselves are vertical angles to  $y$  and  $z$  themselves, then we know that  $y + z = 140^\circ$  also.
11. C. Because  $QT$  and  $RS$  are straight lines, the angle opposite  $y$  is a vertical angle, sharing the same angle measurement of  $y$ . The sum of interior angles of a triangle is  $180^\circ$ . Since there are two triangles, the total sum of all six interior angles would be  $360^\circ$ . We know that two of these six interior angles must be have a value of  $y^\circ$ . Thus, the remaining four interior angles would be  $360^\circ - 2y^\circ$ . Note that you can factor out the 2 from this expression, but you cannot eliminate it entirely. So,  $2(180 - y)$  would be a valid answer choice, but  $180 - y$  would not be.
12. G. Since the arc is part of a circle centered at  $Z$ ,  $WZ = ZV$ , because both are radii. Thus triangle  $WZV$  is an isosceles triangle.  $\angle WZV$  has a measure of  $45^\circ$ , since line  $XZ$  divides the  $90^\circ$  angle  $WZY$  in half. Because this is an isosceles triangle, the remaining angles must be equal, and the angles in a triangle add to  $180^\circ$ , so the two remaining angles add to  $180 - 45 = 135$ .  $135 \div 2 = 67.5$ . Thus  $\angle ZWV$  and  $\angle ZVW$  are each  $67.5^\circ$ .
13. C. Because the trapezoid is isosceles, we know that angles  $W$  and  $X$  are congruent, and angles  $Z$  and  $Y$  are congruent. Since the total number of degrees in any quadrilateral is  $360$ , we know that angles  $W$  and  $Z$  must add up to  $180$ , in which case  $2(q + r) = 360$ .
14. G. A regular polygon is equilateral (all sides are the same length) and equiangular (all angles have the same degree measure). To find the total measure of interior angles in a regular polygon, use the formula  $180(n - 2)$ , where  $n$  represents the number of sides of the polygon. In this case, substitute 8 (since an octagon has 8 sides) for  $n$  to get  $180 \times 6 = 1,080^\circ$ .
15. A. To find the measure of each angle in a regular polygon, use the formula  $\frac{180(n-2)}{n}$ , where  $n$  represents the number of sides of the polygon. In this case, a pentagon has 5 sides, so substitute in for  $180 \times 3 \div 5 = 108^\circ$ .
16. F. In this case, we have a hexagon, which has 6 sides. Substituting 6 in for  $n$  and solving, we find that  $180^\circ \times 4 = 720^\circ \div 6 = 120^\circ$ . Each angle in the regular hexagon has a degree measure of  $120^\circ$ . The question asks for the value of  $x$ , which is a supplementary angle to one of the hexagon's interior angles. Supplementary angles have a sum equaling  $180^\circ$ . Thus,  $x = 180^\circ - 120^\circ = 60^\circ$ .
17. 4. At the intersection of two perpendicular lines, four right angles are formed, since angles around any points sum to 360 degrees.  $360 \div 90 = 4$ .
18. 120. The total number of degrees in any polygon is equal to  $180(n - 2)$ , where  $n$  represents the number of sides in the polygon. A hexagon has  $180(6 - 2) = 720$  degrees, so the average degree measure of each angle is  $720 \div 6 = 120$  degrees.
19. 72. Angle  $AEF$  is an exterior angle of a polygon. In a regular polygon with  $n$  sides, all exterior angles measure  $360 \div n$  degrees. Substituting 5 for  $n$ , we get  $360 \div 5 = 72$ .
20. 232. Quadrilaterals have  $360^\circ$ . In an isosceles trapezoid, there are two pairs of congruent angles. If the base angles are both  $64^\circ$ , then they add up to  $128^\circ$ , so the apex angles total  $360 - 128 = 232^\circ$ .
21. 130. All angles surrounding point  $E$  must add to  $360^\circ$ . Therefore, since  $220^\circ$  are already accounted for, angle  $BED = 140^\circ$ . Since  $ABED$  is also a quadrilateral, it also must have  $360^\circ$ . Angle  $BED$  is  $140^\circ$  and angle  $DAB$  is  $90^\circ$ , so the other two angles must be  $360 - (140 + 90) = 130^\circ$ .

## Measurements

1. C. Since  $AE = AB + BD + DE$ , and we know  $BD = 4$  and  $DE = 7$ , then we know that  $AE = 4 + 4 + 7$ , or 15.
2. G. Count the total number of segments between 5 and 55, as indicated by the tick marks; there are 10 segments formed by the 9 tick marks between 5 and 55. Divide the difference between 55 and 5 (which is 50) by the number of segments.  $50 \div 10 = 5$ . Therefore, each segment represents a separation of 5 units. Since R is 3 tick marks away from 5, we know that  $5 + 15 = 20$ .
3. B. First, determine the scale of the number line by counting the tick marks between two different numbers. Between  $-5$  and  $0$  there are 4 tick marks, one each for  $-1$ ,  $-2$ ,  $-3$ , and  $-4$ . So, each tick mark represents 1 unit. Next, determine the value of P, Q, and R. By counting tick marks, we can see that  $P = -4$ ,  $Q = -2$ , and  $R = 2$ . Lastly, we need only substitute the values into the expressions given to determine that  $P - R$  (or  $-4 - 2 = -6$ ) is the least value. Remember that subtracting a negative results in adding that number. The more negative a number, the less its value.
4. H. Counting the segments between  $-4$  and  $6$ , we can see that the number line is counting by integers. That means A represents  $-5$ , and B represents  $7$ , so their distance is 12. If that distance is divided into 4 equal parts, then each section has a length of 3. This means that L is  $-5 + 3 = -2$ , M is  $-2 + 3 = 1$ , and N is  $1 + 3 = 4$ .
5. D. The distance between point A and point B is 12 units ( $17 - 5$ ). The question describes how C is somewhere along  $\overline{AB}$ , splitting  $\overline{AB}$  into two other segments:  $\overline{AC}$  and  $\overline{CB}$ . If  $\overline{AC}$  is 3 times longer than  $\overline{CB}$ , then we know that the distance from C to B must be  $\frac{1}{4}$  the entire length from A to B. We can let  $x =$  the length of  $\overline{CB}$ . In terms of  $x$ , the length of  $\overline{AC}$  is  $3x$ . So in terms of  $x$ , the total length of  $\overline{AB}$  is  $4x$ . This means  $4x = 12$ , and  $x$  must be 3, if we solve for  $x$ . The location of C must be 3 less than 17, which is 14.
6. E. The number line shown is divided into 5 segments between each integer. This means that the unit of each tick mark is in fifths, or increments of 0.2. We can prove this by dividing the distance between 0 and 1 (which is 1) by 5 (which is  $\frac{1}{5}$ ). Point A is on the 3<sup>rd</sup> tick-mark between 0 and 1. This means it is  $\frac{3}{5}$  of the way between 0 and 1, or 0.6 (which is also  $3 \times 0.2$ ). Similarly, B is on the 2<sup>nd</sup> tick-mark between 1 and 2. This means it is  $\frac{2}{5}$  of the way between 1 and 2, or 0.4 (which is also  $2 \times 0.2$ ). So, B is at 1.4 and A is at 0.6.
7. D. Write down the information provided to help solve this question. By doing so, we can tell that, since the length of  $\overline{WX}$  is 4, and X is located at point 7, then W must be located at point 3. We know the entire length of  $\overline{WZ}$  is 23, and since now we know that W is located at point 3, we know that Z is located 23 units away, at point 26. Since the length of  $\overline{YZ}$  is 6, and we now know that Z is located at point 26, then Y must be 6 less, at 20.
8. F. Write down the information provided to help solve this question. By doing so, we can tell that, since the length of  $\overline{RS}$  is 4.75, that the location of point S is 4.75 from R, which is located at 0.25. This means S is at point 5. The entire length  $\overline{PS}$  is 9.25 long, so we must subtract 9.25 from 5 to determine the location of P.  $5 - 9.25 = -4.25$ . Q is 2.5 units farther to the right (less negative, more positive) than P, so we can add 2.5 to  $-4.25$  to arrive at  $-1.75$ , the location of Q.
9. C. If C is the midpoint of AB, then  $AC = BC = 0.5AB$ . Since we are given  $AC = 17$ , then  $BC = 17$ .
10. F. Draw the information provided in the question. If Q is the midpoint of  $\overline{RS}$ , the two distances (RQ and QS) must be equal and must each represent half of the total distance RS. Therefore, 2 times the



distance represented by QS must equal the entire length of RS, since QS by itself represents half of the entire length RS.

11. B. The midpoint between two points on a line can be calculated by taking the sum of both points and dividing by 2. In this case, we have  $-4 + \frac{3}{4} = -3\frac{1}{4}$ . Divide this by two, we arrive at  $-1\frac{5}{8}$ .
12. H. The best way to tackle this question is to draw the information provided. If X is the midpoint of AB, then  
 $AX + XB = AB$ , and  $AX = XB$ . If M is the midpoint of XB, then  $XM + MB = XB$ , and  $XM = MB$ . Because of this,  
 $XM = MB = \frac{1}{2} \overline{XB}$ . However, the question asks for AM. Since  $AX = \frac{1}{2} \overline{AB}$ , then  $AX + XM = \frac{3}{4} \overline{AB}$ .
13. C. If  $2QR = RS$ , then we know that QR is exactly half of RS. If T is drawn on the line between R and S, and  $RT = TS$ , then T is located at the midpoint of RS. This means that  $QR = RT = TS$ . Each of these 3 segments along QS is equally long. If  $QS = 36$ , then  $36 \text{ units} \div 3 \text{ segments} = 12 \text{ units per segment}$ .  $QT = QR + RT$ , so  $QT = 24$ .
14. G. If WZ is 16 units apart, then W is located at  $13 - 16 = -3$ . Since X is the midpoint of WY, then  $WX = XY$ . Since  
 $WX = -1 - (-3) = 2$ , then  $XY = 2$ , and Y is located at 1. The midpoint of YZ =  $13 + 1 = 14 \div 2 = 7$ .
15. B. 1 donut can be glazed in 1 second. To glaze 36,000 donuts would take 36,000 seconds. In hours, this is  $36,000 \text{ seconds} \div 60 \text{ seconds per minute} \div 60 \text{ minutes per hour} = 10 \text{ hours}$ .
16. F. We can solve this question algebraically. Let  $x$  equal the distance in miles of Yunru's afternoon commute by highway. In terms of  $x$ , her morning commute by backroad is  $x + 7$  miles long. In total,  $x + x + 7 = 57$ . Simplify by combining like terms:  $2x = 50$  and solve for  $x$ , which equals 25. Note that 25 is the distance by highway, and the question asks for the morning commute distance, which is by backroad. Thus,  $25 + 7 = 32$ .
17. B. First, calculate the area of the walls of the room. If the bedroom itself is 10-foot by 12-feet, that means 2 walls have a length of 12, and the other 2 have a length of 10. Each wall is 8 feet tall. So the two walls that have length 10 feet have an area of  $8 \times 10 = 80 \text{ sq. ft.}$  each, and the two walls that have length 12 have an area of  $8 \times 12 = 96 \text{ sq. ft.}$  each. In total, the square footage of the walls is  $160 + 192 = 352$ . Since 1 gallon of paint only covers 300 sq. ft., then one would need 2 gallons of paint.
18. G. If the total area of Figure X = 48 sq. in., and there are 16 individual small squares, then each small square equals  $48 \div 16 = 3 \text{ sq. in.}$  In Figure Y, there are also 16 small squares, each 3 sq. in. in area. Thus, the area of Figure Y is also 48 sq. in.
19. A. First, convert the measurement of the wall from feet to yards. Since there are 3 feet per yard, we divide each measurement in feet by 3 to arrive at 6 yards and 8 yards. The area in square yards is simply  $6 \times 8 = 48$ . Note that this is not the same as calculating the square footage then dividing by 3.
20. H. The volume of a box can be measured by multiplying the length by width by height. In this case,  $2 \times 3 \times 5 = 30 \text{ cubic meters}$ . For every cubic meter, there 0.4 kg of packing foam. So:  $30 \times 0.4 = 12$ .
21. B. The total volume of the pool is can be calculated by length times width by height (in this case, depth). Since it is a square, we know that the sides are all 10 m long. Thus,  $10 \times 10 \times 3 = 300 \text{ cubic meters}$ . More water per minute is being drained from the pool than is being added to the pool:  $8 \text{ cubic meters per minute} - 3 \text{ cubic meters per minute} = 5 \text{ cubic meters per minute of water being drained}$ . If the pool started at  $\frac{2}{3}$  full, then it is 200 cubic meters full. It will take 200 (cubic meters)  $\div 5 \text{ (cubic meters per minute)} = 40 \text{ minutes to fully drain, at 12:40 p.m.}$
22. F. Since there are 60 minutes per hour, we need only multiply:  $3.65 \text{ hours} \times 60 \text{ minutes per hour} = 219 \text{ minutes}$ .



23. C. One full cycle consists of 90 minutes where Samson runs for 45 minutes then walks for 45 minutes. The question asks at what time the fourth period of walking will begin. This means 3 full cycles and 1 half cycle will be completed by this time.  $3 \text{ full cycles} \times 90 \text{ minutes} = 270 \text{ minutes}$ . Add to this 45 minutes running (for the remainder half cycle) and the total elapsed time is 315 minutes. Since there are 60 minutes per hour, the elapsed time is  $300 \text{ minutes} \div 60 \text{ minutes} = 5 \text{ hours}$ , with 15 minutes remaining. Thus, 5 hours and 15 minutes after 9:00 a.m. is 2:15 p.m.
24. F. We know that there are 24 hours in a day.  $70 \div 24 = \sim 2.9$ . So we know that 70 hours after 12:30 p.m. on Monday will be very close to (but 2 hours short of) 3 full days (72 hours) later. Three full days after Monday at 12:30 p.m. would be Thursday at 12:30 p.m. However, only 70 hours, not 72 hours, have elapsed. Instead of a full 3 days, we can count back 2 hours to 10:30 a.m. on Thursday.
25. B. If Byron biked 3 times the distance of Amanda, then he biked a total of  $22 \times 3 = 66 \text{ miles}$ . If he did so in 2 times the amount of time as Amanda, then he biked this distance in  $3 \times 2 = 6 \text{ hours}$ . In miles per hour, this equals  $66 \text{ miles} \div 6 \text{ hours} = 11 \text{ miles per hour}$ .
26. H. The difference in hours between EST and HAST is 1:00 P.M. – 8:00 A.M. = 5 hours. If the first plane arrived in Honolulu at 6:00 P.M. HAST, then the EST equivalent of its arrival in Honolulu is 11:00 P.M. EST. This means that the total flight time was 12:00 P.M. EST – 11:00 P.M. EST = 11 hours. Following this same amount of flight time, the 7:00 A.M. HAST flight from Honolulu would arrive 11 hours later in Newark, at 6:00 P.M. HAST. 6:00 P.M. HAST + 5 hours = 11:00 P.M. EST.
27. –3.5. The best way to solve is to draw a diagram of a number line with W, Y and Z at –12, –5 and 8, respectively. Draw A between X and Y, and draw B between Y and Z. Point A is the midpoint of XY, so it is at –8.5, and point B is the midpoint of YZ, so it is at 1.5. Their midpoint is then –3.5.
28. 8. Point P is at –4, point Q is at 8, and point R is at 24. The midpoint of PQ is then 2, and the midpoint of PR is 10, so the distance between them is 8.
29. 4. J is at –8 and K is at 0. It is 4 tick marks from J to K, so  $\frac{1}{4}$  of the way would be at –6. L is at 14, so the midpoint of A and L is 4.
30. 486. 2 yards = 72 inches, and 3 yards = 108 inches. If each tile is 4 inches long, it will take 18 tiles to run the length of the bathroom, and 27 tiles to run the width, requiring a total of  $18 \times 27 = 486$  tiles.

## Coordinates

1. A. Point A is in the first quadrant, where all  $x$  and  $y$  values are positive. Point B is in the third quadrant, where all  $x$  and  $y$  values are negative. The only choice with negative  $x$  and  $y$  values is represented by  $(-x, -y)$ .
2. H. Point A is 12 units along the  $x$ -axis, and 5 units along the  $y$ -axis. This means segment OA actually represents the hypotenuse of a triangle with a height drawn from A up to the  $x$ -axis or a leg drawn from A over to the  $y$ -axis. It is a special right triangle with legs 5 and 12. This means its hypotenuse must be 13. The question tells us that OB is  $2 \times OA$ , so this is  $13 \times 2 = 26$ .
3. C. Since the figure is a rectangle, we need only to determine the length of two sides with different lengths. We know that (3, 3) and (3, 1) are 2 units apart on the  $y$ -axis, but aligned on the  $x$ -axis. Similarly, we know that (3, 1) and (0, 1) are aligned on the  $y$ -axis, but are 3 units apart on the  $x$ -axis. Therefore, the length and width of the rectangle are 2 and 3. So, the area is 6.
4. H.  $y$  values are graphed on the vertical axis. We can tell if more than one point in a graph shares a  $y$ -value with another point by visually scanning each graph. Looking left to right, any time we see more than one point at the same vertical distance from the origin (the intersection of the  $y$  and  $x$  axes), those points have the same  $y$  value. This is the case in all graphs but H.
5. D. There are infinite points that are exactly 2 units from the origin. Drawing them all creates a circle about the origin with radius 2.

6. F. Turning  $(j,k)$  into  $(-k,j)$  means switching the order of the numbers and negating what was originally the  $y$ -coordinate.
7. A. The two given vertices create a line segment with a length of 6 units. In any triangle, any two sides must add up to more than the third side. This means that the two other sides must add up to more than 6, so the perimeter must be more than 12.
8. G. A proportional relationship here means that the  $x$  and  $y$  coordinates should be in the ratio 2:1. Only  $(14,7)$  has this relationship.
9. A. The first two vertices are 5 units apart, so every side length should be 5 since it's a square. When a square on the coordinate grid is perpendicular to the axes, each coordinate point should appear twice. There are two 2's and two 3's, meaning the  $x$ -coordinate is 7 and the  $y$ -coordinate is  $-2$ .
10. H. There are four possible squares that can be drawn from the description. The other vertex on the  $y$ -axis is either at  $(0,5)$  or  $(0,15)$ . Either way, all vertices are in the top quadrants, so all coordinates must be positive.
11. 24.5. The right triangle has a height of 7 and a length of 7, so the area is  $\frac{1}{2}bh$ , or  $\frac{1}{2}(7)(7) = 24.5$  square units.
12.  $-3$ . Since the given endpoint of the diameter is 3 units to the right and 4 units up, the other endpoint must be 3 units to the left and 4 units down, making it  $(-2, -3)$ .
13. 15. Drawing a rough sketch shows that the triangle has a base of 5 and a height of 6. The area of a triangle  $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(5)(6) = 15$ .
14. 2. If  $(4,-2)$  is reflected over the  $x$ -axis, the new points are  $(4,2)$ . If it's then reflected over the  $y$ -axis, its new coordinates are  $(-4,2)$ .
15. 16. The trapezoid has a bottom base of 6, a top base of 2, and a height of 4. The area of a trapezoid is  $(\text{average of the bases}) \times (\text{height}) = 4 \times 4 = 16$ .
16.  $-6$ . The graph shows a line that goes through the origin. Find  $-2$  on the  $x$ -axis and trace down to the line shown. The  $y$ -axis value at this point is  $-6$ .
17. 1.5. The midpoint of a line segment can be found from the average of its endpoints. Since the  $y$ -coordinates of the endpoints are 5 and  $-2$ , the midpoint must have a  $y$ -coordinate of  $5 + (-2) = 3 \div 2 = 1.5$ .

## Final Practice Test (Form B)

### Part One: English Language Arts

1. A. *Subordination & Coordination*. This sentence is made up of two different, complete thoughts. The first is "Jessica was...park." The second is "She was...sickness." We can combine these sentences together to form one larger sentence using the appropriate conjunction, but we must pay attention to the relationship between the two different ideas. In the first, we are told that Jessica is excited to ride an "extreme rollercoaster." In the second, we are told that she is "deathly afraid" of qualities one normally associates with rollercoasters (heights, motion). Therefore, if she is excited about the roller coast, it must be "in spite of" her fears. In other words, she is excited "even though" she has fears, not "because of" or "since" she has fears. The solution results in an independent clause ("Jessica was...park") followed by a subordinating conjunction and clause ("even though...sickness"). The subordinating clause tells us more information about Jessica's excitement: that she was excited though she had fears.
2. G. *Unnecessary Punctuation*. Though multiple adjectives describing the same noun should be separated with commas, there is no need to separate the last adjective from the noun with another comma. In this case, "season" is described as "cold" and "rainy." "Rainy" doesn't need to be separated from season by a comma.

3. *C. Subordination & Coordination.* The first two choices leave out important information: that the grapefruits and carrots are Floyd's favorite fruits and vegetables, respectively. When we use the word "respectively," we must be careful that the reference words are aligned in order. In this case, grapefruits are fruits, and carrots are vegetables. Since "grapefruits" was listed before "carrots," then so too must we list "fruit" before "vegetable."
4. *G. Organization & Logic.* Sentence 3 describes how people find qualities they like about plants or animals and increase the number of those things. The first choice is imprecise since it describes how people multiply organisms—a strange phrase. In addition, "they" is vague—is it referring to people, or organisms? "It," in the second choice, is also vague. Is the sentence referring to selective breeding, or changing traits in a laboratory? Saying "some people" and "things that they enjoy" is also vague, and helping those things "become part of the plant in the future" is awkward and vague. Instead, the third choice clearly describes what the sentence refers to (the latter being selective breeding), and what it is.
5. *A. Organization & Logic.* Sentence 17 brings a different point of view to the passage. Where the preceding paragraph described many benefits, the last paragraph describes reservations and drawbacks. Therefore, we should use a transition word like "nevertheless," which signals a contrast, rather than transitions like "accordingly" and "likewise," which signal a similarity. "Specifically" would be used to expand further upon the ideas in the preceding sentence/paragraph.
6. *H. Organization & Logic.* Sentence 4 describes how, over time, organisms like plants develop qualities that people like. Only the last choice describes things people "love" and how they happen.
7. *B. Organization & Logic.* Sentence 5 describes how people can change the traits of organisms quickly thanks to science. Sentence 6 describes how science has also enabled people to combine genes across different domains. The two are related, and can be combined with a "but...as well," which is similar to "but...also." The other choices are nonsensical, using words like "though" and only "but." The first choice changes the meaning of the sentence.
8. *H. Grammar.* The noun "genes" is plural, so the verb "to be" should be plural as well. This is "were." "Insect-killing" is properly hyphenated as a compound adjective describing "toxins." The comma after 1995 would separate the sentence in a way that creates fragments on either side of it. The sentence is written in the past tense, so changing "was not" to "is not" is erroneous.
9. *D. Organization & Logic.* Sentence 16 lays out several advantages to using GMOs. Only the last choice describes a specific example of "higher nutritional value." The other choices describe drawbacks to the use of GMOs.
10. *E. Organization & Logic.* The passage is concerned with describing what GMOs are and how they are beneficial, as well as the negative aspects. Describing more about bacteria and archaea is not important to the main idea. Sentences 11 and 14 are important to describing Bt, the central example of GMOs in the passage.

Mary Oliver

11. *B. Main Idea.* The passage shares how Oliver began writing as a child while walking in the woods (paragraph 5), and how her work influenced the masses (paragraph 2). While her poems are known for being emotional, poetry was about empowerment and rising above darkness for Oliver (paragraph 4). And though she was famous – Oprah Winfrey created a special issue of O magazine for her, highlighting a "rare interview that Ms. Oliver gave" – someone that wrote for popular appeal would seek interviews, rather than rarely giving them.
12. *G. Craft & Structure.* Prior to this sentence, the passage details her popularity, publications, and honors. After this sentence the passage explores how nature inspired her to write and how combining the two "was [her] salvation" (paragraph 5). This sentence serves as a transition between the two topics. She spent time in the woods in both her old age and in her youth, so there is no contrast.



13. A. *Craft & Structure*. Doorways are passages into other rooms. A closed door doesn't share what is behind it; therefore, by Oliver referring to a "thousand opening doors," she illuminates the unknown world by opening them through poetry. Even though poetry was Oliver's chosen artistic medium, prior to the door statement she states that she "did not think of language as the means to self-description" or self-exploration, and the doors open "past" herself.
14. H. *Supporting Idea*. The idea in paragraph 4 that Oliver found herself in the woods is echoed by the statement in paragraph 5 about finding her voice in the woods. This is figuratively true, as she found ideas for poetry in the woods, and used it as a way to express her thoughts, confusion, and concerns. The other choices merely describe the reasons why Oliver wished to go into the woods.
15. C. *Craft & Structure*. Here the reference to cathedrals elicits connotations of comfort and wonder, given the words "shelter" and "spiritual renewal." Her desperation is described in an earlier sentence.
16. E. *Supporting Idea*. Paragraph 6 guides the reader into one of Oliver's poems, explaining how her poetry reflects what she sees in nature. The first quote draws on this idea, and combines it with her personal history to show how poetry and nature were her ways to transform pain into beauty. The other quotations are supporting details, and do not describe the central idea.
17. D. *Inference*. Given the author's positive tone about Oliver (paragraph 5) despite the snubs of "some critics," we can infer that the author believes poets should follow their own interests and not the expectations of others. Even though the author shares Oliver's writing in old age, that was a fact about Oliver and not a main focus of the passage.

Adapted from *The Youngest Girl in the Fifth*

18. F. *Craft & Structure*. The simile offers more detail to how Gwen "snatched back her exercise book." The author compares Gwen's motions to "a mother clutching her first-born" to show how deeply invested she is in her work. It may be that Gwen's essay is of very high quality but this quote does not mention the envy of others.
19. A. *Inference*. In paragraph 13, Netta uses examples of how other people cheat, or claim credit for others' work, to seal her argument. Gwen disagrees, and appears to be hurt by Netta's reference to unethical clergymen, but Netta presses her case regardless. Netta has the opposite of a strong sense of fairness. She only cares about getting her way, and will use any justification to get it.
20. G. *Supporting Idea*. At this moment, Gwen makes the critical and difficult decision to accept Netta's offer. Up until this point, Netta has been trying to convince Gwen to make a deal (essentially, blackmailing Gwen). After this point, the excerpt describes the consequences of that action: Gwen is miserable. Netta would indeed rather cheat than work hard, but this is supported in other paragraphs, not paragraph 24.
21. C. *Supporting Idea*. This quote shows that Gwen is upset at giving away her essay because she wants to be "highly commended." Though paragraph 19 implies that Gwen wants to avoid the consequences of breaking the china, the consequences will be punishment rather than merely being badly perceived.
22. H. *Inference*. In paragraph 15, the excerpt describes Netta's determination to get something – money, or something else – so she blackmails Gwen. Paragraph 16 describes how Netta could be "uncommonly nasty," which factors into Gwen's decision to ultimately give Netta what she wants. The fact that Netta is "neither brilliant nor a hard worker" isn't a reason why Gwen cooperates with Netta—it's a reason why Netta decides to blackmail Gwen for the essay.
23. A. *Main Idea*. As a result of her agreement with Netta—motivated by Gwen's desire to hide the truth—Gwen is forced to write a second essay. Had Gwen faced the consequences of her actions, she may have gotten in trouble with Miss Roscoe but wouldn't have had to give her original essay away.



24. *G. Supporting Idea.* The lines in question show that Netta could not have otherwise gotten a chance to receive praise for her work. The fact that she is not known for being the best student is echoed by the students' incredulous reaction to the announcement that Netta wrote the best essay.
25. *D. Craft & Structure.* Gwen is filled with frustration because she is witnessing someone else – someone she dislikes – getting the credit she had wanted so badly for working hard on her essay, and she cannot voice the truth. There is no evidence that she is worried about people finding out about her arrangement with Nettie.

### The Big One

26. *E. Main Idea.* The passage discusses the events of the San Francisco earthquake of 1906, and the extent of the damage it caused. This answer option identifies the topic, and describes the quake as “one of the most powerful...ever recorded.” Although the passage states facts about the fires that occurred due to the quake (paragraph 4), the fires are a detail rather than the central idea. The passage mentions, but is not primarily about, how San Francisco enjoyed decades of peace after the quake.
27. *D. Supporting Idea.* The map highlights the San Andreas Fault's main fracture, and San Francisco's location squarely on top of it, highlighting the main claim of paragraph 1. The map does not illustrate any “man-made factors” that caused the quake or population numbers.
28. *H. Inference.* In paragraph 3, the author states that the quake happened before the development of the Richter scale, and its magnitude was estimated based on later evaluations of the damages and changes it caused. We may infer then that this is not as accurate as having the technology to measure it in present time. The author repeatedly marvels at how short the actual quake was (paragraphs 2 and 8), so the damages were not caused by the sustained length of the quake.
29. *B. Craft & Structure.* At the beginning of paragraph 2 the author first mentions “violent shocks” lasting only seconds, then the “rupturing” miles, and finally the “24 feet slip,” building the stress and fear felt by the people. Although the paragraph “gives a scientific overview,” it is not intended to bore the reader. In addition, the use of a quote at the end of the paragraph drives home the feeling of being there.
30. *G. Inference.* Paragraph 4 describes how the firefighters tried creating “firebreaks by demolishing buildings.” They believed that “sacrificing some buildings” would “stop the fires from consuming” more, but the inexperienced firefighters “started more.” The statement “caused additional damage [...] in efforts to solve the problem” best describes the firefighters' experience.
31. *A. Craft & Structure.* The author's word choices in the phrase “even knowledge” shows the relentlessness of the fire, and the word “discriminate” implies intelligence or ability to choose, which personifies the fire. Although the sentence personifies the flames, it does not imply the flames were capable of intention.
32. *F. Supporting Idea.* This option shows how the House and Senate committees, which are part of the government, sent victims the things they needed. In the previous option, the officials are shown to be identifying, or seeing, the problem, but their action (“passing out clothing”) does not match the need of the peoples (“food and shelter were the main concerns”).
33. *C. Craft & Structure.* In the passage, the events of the 1906 San Francisco earthquake are told in narrative order, and paragraph 9 brings the passage to a close by reaffirming the reader that “San Francisco recovered” and “enjoy[ed] decades of geological peace.” Although the paragraph mentions geography, it doesn't emphasize the destruction, which occurred in previous paragraphs.

### Learning Differently

34. *F. Main Idea.* The passage is primarily concerned with explaining why rote memorization is not as helpful for students as conceptual understanding, despite memorization's role in helping students perform well on standardized tests. The passage offers reasons and explanations supporting the notion that rote memorization does not help students in problem-solving unfamiliar situations, such

as those encountered in the workplace, for example. The passage does not say that rote memorization can lead to critical thinking skills, and actually implies the opposite.

35. A. *Inference*. In the second paragraph, the author writes that rote memorization is an ineffective learning tool because it lacks context for the information memorized, and does not develop conceptual understanding. The statement provides an example of how memorizing information (i.e. a formula) does not equal understanding it. It is not a critique of weak math training.
36. F. *Craft & Structure*. The word “pitfall” means “an unsuspected or hidden danger.” Paragraph three reveals the inadequacies of rote memorization in real learning, contrary to what people expect. Therefore, the use of “pitfalls” to describe rote memorization suggests it has unexpected drawbacks for students. The “dangers” of rote memorization are not obvious, since initially and on tests, it seems to lead to student success.
37. A. *Inference*. The US Department of education study reinforces the idea that high test scores do not correlate to later life success, which is the main claim of paragraph 4. Though the study itself discusses other countries, the paragraph is specifically concerned with education in America, and not how the problem affects children in other countries.
38. G. *Inference*. This statement helps explain how human minds and computers are designed to do different things, and how human minds are ultimately more suited to “process complex information.” Though rote memorization is more like how a computer processes information, the passage does not suggest that memorization can make the human brain more like a computer.
39. B. *Craft & Structure*. Paragraph 5 introduces the notion of developing critical thinking skills in students, in lieu of encouraging mere rote memorization. Paragraph 6 elaborates on this idea by citing the long-term benefits of critical thinking skills in students’ learning, therefore reinforcing its importance. Critical thinking is contrasted with rote memorization, but no mention is made of critical thinking’s short-term benefits.
40. H. *Supporting Idea*. The notion that idea-sharing (mentioned in paragraph 9 as one type of exercise to develop critical thinking skills) “teaches students how to learn and work in a social setting, which workplaces require” supports the claim that critical thinking skills are useful in life beyond school. Though the other options state that critical thinking skills are beneficial, they do not show specifically how these skills can be helpful in life after school.
41. C. *Craft & Structure*. Paragraph 9 tells us that brainstorming “can reveal seemingly crazy but often creative and novel ways to tackle problems.” This highlights the positive, albeit unexpected outcomes of one type of critical thinking exercise. “Crazy” here is used in a positive, not a negative, sense.

“Summer”

42. H. *Inference*. The first several lines describe the relationship between “some” people and nature. To “some,” nature is a “closest friend” who they “hold dear communion with.” This “dear communion” means that nature benefits man and can be interpreted as the two coexisting peacefully. The speaker does not compare the relationship to a curse, only describing how it benefits people. The fact that nature is beautiful is given, but this by itself does not develop the central idea of the poem. The speaker uses “some” to show that not everyone can take advantage of this relationship.
43. C. *Craft & Structure*. The word “voice” tells us that the waters are compared to a person. Notice the preceding and following lines, as well, which describe the hills as one with which people can “hold dear communion” and the winds as bringing “healing in their sound.” These are things that only people can do. All of these are positive, not harmful. These do not suggest that people can gain inspiration or advice from them or overcome weaknesses. Instead, this suggests comfort and relief, as mirrored by the word “soothes.”

44. F. *Craft & Structure*. This metaphor is very powerful. The city, a place where many people live close together, is cast in a negative light, as a “prison house.” The speaker doesn’t personally hate towns and cities; in these lines, she is describing how people generally perceive summer and cities.
45. A. *Craft & Structure*. It’s important to read these lines in context. In this part of the poem, the speaker describes how he agrees with other people about some aspects of summer: nature is beautiful and can do a lot. But line 18 shows there is more that nature can provide, which is described later in lines 34-42. Nature is not described as wild and untamable but as an ally that helps people.
46. F. *Supporting Idea*. Lines 11-12 describe how other people feel, not how the speaker’s views differ from those feelings. Lines 18-19 show that the speaker agrees with the idea that nature is beautiful but has deeper thoughts that go beyond nature – that there is beauty to be found amongst people. This is especially true because the lines follow the description of nature’s beauty. Lines 20-21 and 25-26 describe an idyllic scene that others would agree with.
47. D. *Supporting Idea*. Earlier in the poem, the speaker describes how nature is beautiful and provides inspiration and relief, but that there is more to be found elsewhere. This is what is meant when the speaker says “I love the very human heart of man” (note the preceding line: But more than these, and much, ah, how much more). The answer to this is in lines 35-36, where the speaker describes more of what this means: close contact with others (like living in a city) is “like a lantern shining in the night” – a comforting and reassuring image.
48. G. *Main Idea*. These lines show a distaste for the season of summer, with negative words like “void,” “lags,” and “silent.” They show that the poet, unlike most people, does not see summer in a completely positive light. Although it mentions how inspiration lags in summer, she is using that as a detail to make her point, and it is not the central idea of the poem.
49. C. *Inference*. To the speaker, summer is a pause, a time for respite – a time when “feeling sleeps.” It is not a place or time where “beauty dwells not” – the speaker gives many examples of summer’s beauty. And yet, summer does not bring the healing, for the speaker, as it does for other people. The time to gain new experience is during the winter. As you can see, understanding the context in which the line is written is extremely important.
50. E. *Inference*. The lines toward the end of the poem describe how winter has benefits because it is difficult to be in nature so it is better to spend time with other people. These sentiments are echoed in lines 35-36 and lines 41-42. They are contrasted by lines 11-12, when the speaker describes how some people believe that winter is dreary, and they only endure it for the sake of summer.

Superheated

51. A. *Main Idea*. The passage is about the life cycle of the sun, starting from when it was first formed 4.5 billion years ago (paragraph 2) to what it will eventually become (paragraphs 5-6). Although it explains nuclear fusion and the amount of energy given off by the sun, those are supporting details, not the central idea.
52. G. *Supporting Idea*. Paragraph 2 states: “Nuclear fusion has sustained the sun ever since, resulting in the release of massive amounts of energy.”
53. D. *Supporting Idea*. Paragraph 5 discusses how the sun grows in size: “Every 500 million years or so, the sun will double in size until it reaches a size more than 200 times larger than it is today.” There is no mention of how much energy other stars output, so we cannot infer that the sun’s energy is less than other stars in our galaxy. And while the passage discusses how the Earth’s atmosphere absorbs the sun’s energy, it does not state whether it absorbs almost all or some of that energy, nor that that energy goes to powering communication efforts specifically.
54. F. *Supporting Idea*. The sun will become a red giant once the hydrogen runs out (paragraph 5). After it runs out of helium, it will become a white dwarf star.



55. C. *Supporting Idea*. All of the options are true, except that the sun will not begin burning off its supply of hydrogen; it will have already done so, by the time it becomes a red giant.
56. E. *Supporting Idea*. The sun is currently a yellow dwarf star (paragraph 1), will become a red giant after burning through its hydrogen (paragraph 5), and will eventually become a white dwarf star (paragraph 6) after its helium supply is exhausted.
57. A. *Supporting Idea*. The table shows the mass of each celestial object in our galaxy, and demonstrates how the sun dwarfs all other objects. This supports the claim “The sun alone accounts for more than 99% of all matter in our solar system” (paragraph 1). The table does not show information about the radiation from the sun, nor how its size will change in the future.

**Part Two: Math**

58. 1.5. *Numbers & Operations—Ratios & Proportions*. We have been given two ratios, one of which can be simplified: if 2 jorps = 4 saffs, then 1 jorp = 2 saffs. Now we know that 1 jorp is equal to both 3 wims and 2 saffs, meaning 3 wims = 2 saffs. We simply divide both sides by 2 to get 1 saff = 1.5 wims.
59. 120. *Numbers & Operations—Counting Principle*. Since 5 students are getting in line, there are 5 options for the first position, 4 remaining options for the second position, 3 options for third position, 2 for fourth position, and only one left for the last position. The answer is  $5 \times 4 \times 3 \times 2 \times 1 = 120$ .
60. 0. *Algebra—Inequalities*. The first statement says  $x$  must be greater than  $-2$  and less than  $4$ , so  $x$  can be  $-1, 0, 1, 2$ , or  $3$ . The second statement says  $x$  must be greater than  $3$  and less than  $10$ , so  $x$  can be  $4, 5, 6, 7, 8$ , or  $9$ . There are no numbers that satisfy both statements.
61. 7. *Geometry & Measurements—Measurements*. Working backwards, if the length of DB is 3, then the length of CB must be 6 and the length of AB must be 12. If B is 12 units to the right of A, then it is  $-5 + 12 = 7$ .
62. 4. *Numbers & Operations—Word Problems*. There are  $35 \times 2 = 70$  orange candies,  $4 \times 15 = 60$  green candies, and  $60 + 70 = 130$  purple candies. In total, this is 260 candies. Divided among 65 party bags, this would mean there are  $260 \div 65 = 4$  candies per party bag.
63. A. *Numbers & Operations—Factors, Multiples, Exponents, & Radicals*. First add up the numbers under the radical. We get the square root of 25. The square root is essentially the opposite operation of raising a number to the second power (also called “squaring” that number). That means we are trying to find a number that we can multiply by itself to get 25. The answer is 5 because  $5 \times 5 = 25$ .
64. E. *Algebra—Plugins*. The fact that Adina is older than Boaz can be represented by  $a > b$ . The fact that Adina is younger than Ezra can be represented as  $a < e$ . By combining the two inequalities, we can see that  $b$  must be least,  $e$  the greatest, and  $a$  in the middle. Therefore,  $b < a < e$ .
65. B. *Numbers & Operations—Operations*. Notice that all answer choices use the same digits, and only the decimal place changes. We can use logical reasoning to determine that  $324 \div 72 = 4.5$ . Since the decimal in 0.324 is two places from where it should be, we know we must move the decimal in 4.5 two places to the left, giving us 0.0045.
66. F. *Algebra—Algebraic Expressions & Equations*. One way to solve is to recognize that the value we are looking for,  $2m - 10$ , is exactly half of the left side of the given equation,  $4m - 20$ . That means, instead of solving for  $x$ , we can simply divide the equation by 2, giving us  $2m - 10 = 4.5$ .
67. A. *Geometry & Measurements—Measurements*. The midpoint between A and B is  $(3 + (-5)) \div 2 = -1$ . The midpoint between A and C is  $(7 + (-5)) \div 2 = 1$ . The distance between the two midpoints is  $1 - (-1) = 2$ .
68. H. *Numbers & Operations—Word Problems*. Each notebook originally costs  $\$7.50 \div 3 = \$2.50$  each. If we add  $\$0.75$  to this, we get a price of  $\$3.25$  each. 5 notebooks would cost a total of  $\$3.25 \times 5 = \$16.25$ .



69. A. *Numbers & Operations—Factors, Multiples, Exponents, & Radicals*. Resolve each exponential term so that the equation becomes  $\frac{16+9}{-8-1} = -\frac{25}{9}$ .
70. H. *Algebra—Algebra in Context*. If  $x$  = the number of miles Bob biked, then  $3x$  equals the number of miles Alex biked, in which case they biked a total of  $4x$  miles. Since we know the total is 72, our equation is  $4x = 72$ , so  $x = 18$ . But the question is asking for Alex's miles, which is  $3(18) = 54$ .
71. C. *Numbers & Operations—Factors, Multiples, Exponents, & Radicals*. The greatest common factor is 105. Since the greatest common factor cannot be greater than the two numbers, we can eliminate 4,620 immediately. We must test the other answer choices individually. All of them are factors, but of them, 105 is the greatest.
72. E. *Numbers & Operations—Ratios & Proportions*. Since 1.2 euros is equal to 1 dollar, and 102 yen is also equal to 1 dollar, then 1.2 euros = 102 yen. To solve for 1 euro, we need only divide both sides by 1.2. Doing so results in 85 yen.
73. C. *Geometry & Measurements—Triangles*. An equilateral triangle has sides that are the same length. A regular pentagon is equilateral, meaning all of its five sides are the same length. If one side of the triangle has length 5, and all sides are the same length, then the perimeter of the triangle is 3 sides  $\times$  5 = 15. The pentagon has 5 equilateral sides, so each side is  $15 \div 5$  sides = 3. The question asks for the length of 3 sides, so  $3(3) = 9$ .
74. H. *Algebra—Algebraic Expressions & Equations*. To solve, start by combining like terms:  $4 - 2x = 14 - 7x$ . Then make sure the variable is only on one side of the equation by cancelling out the smaller variable, here by adding  $7x$  to both sides:  $4 + 5x = 14$ . Isolate the variable by cancelling out the constant and then dividing by the coefficient:  $5x = 10$  and  $x = 2$ .
75. B. *Numbers & Operations—Numbers*. 3 fits into 100 approximately 33.333 times, so there are exactly 33 positive multiples of 3 that are less than 100. 3 fits into 50 approximately 16.667 times, so exactly 16 of those 33 multiples are less than 50, so there are 17 multiples of 3 between 50 and 100.
76. G. *Numbers & Operations—Word Problems*. If they want 60% of their sales to be burgers, that means that 40% of their sales will be hotdogs. The number of hotdogs remains the same at 100, which means that 100 should be 40% of the total number of sales. If  $x$  represents the total number of both hamburger and hotdog sales, then we can find  $x$  using the equation  $100 \div x = 40 \div 100$ . Cross-multiply to get  $40x = 100 \times 100$ . Solving for  $x$  we get  $x = 250$ . This is the total number of sales. Since they sold 100 hotdogs, the number of burgers they need to sell is  $250 - 100 = 150$  burgers. They have already sold 100 burgers, so the number of additional burgers they need to sell is  $150 - 100 = 50$  additional burgers.
77. D. *Probability & Statistics—Probability*. Let  $x$  represent the probability of choosing a black bead out of the box. In terms of  $x$ , the probability of choosing a blue bead is  $3x$ . This means the ratio of blue to black beads is 3:1, which we can also represent as fractions. The chance of drawing a blue bead is  $\frac{3}{1+3}$ ; the chance of drawing a black bead is  $\frac{1}{1+3}$ . If there are 12 beads, then  $\frac{3}{4} \times 12 = 9$ .
78. G. *Numbers & Operations—Ratios & Proportions*. If  $\frac{3}{7}$  of K is 210, then each  $\frac{1}{7}$  of K is  $210 \div 3 = 70$ , in which case  $\frac{5}{7}$  of K is  $5(70) = 350$ . We can rule out 150 because it is less than 210.
79. B. *Numbers & Operations—Ratios & Proportions*. Students must be familiar with the metric system. 1 gram is equal to 1,000 milligrams. Setting up a proportion, we get  $\frac{1,000 \text{ milligrams}}{1 \text{ gram}} = \frac{90 \text{ milligrams}}{x \text{ grams}}$ . Cross multiply and solve for  $x$ :  $90 = 1,000x$ , so  $x = 0.09$ .
80. E. *Numbers & Operations—Word Problems*. If there are  $s$  students in total but  $n$  students are not going, then they are taken out of the total, leaving  $s - n$  students who must pay for the whole trip. The amount each of them must pay is equal to the total amount divided by the number of students going.

81. *A. Numbers & Operations—Word Problems.* This is a remainder problem. Dividing the total number of bagels by the number of bagels on each tray, we get  $163 \div 18$ , which results in 9 remainder 1. That means 9 trays will be filled to capacity and there will be 1 bagel left to go on a tray by itself.
82. *G. Geometry & Measurements—Area & Perimeter.* Since MNOP is a rectangle, then  $PO = MN = 4$ . The perimeter is 14, so sides  $NO + MP = 14 - 8$ . This means NO and MP are each  $6 \div 2 = 3$ . We have two a special right triangles with sides 3-4-5, meaning the hypotenuse NP is 5.
83. *A. Geometry & Measurements—Angles.* The distance between parallel lines is constant, so they will never intersect.
84. *G. Probability & Statistics—Averages.* We can solve this algebraically.  $\frac{5 + 5 + 8 + 10 + x}{5} = x..$   
Simplify by combining like terms and canceling denominators:  $\frac{28 + x}{5} = x$ . becomes  $28 + x = 5x$ .  
Isolate the variables to one side of the equation and simplify:  $28 = 4x$  and  $x = 7$ .
85. *C. Geometry & Measurements—Circles.* The area of a circle is equal to  $\pi r^2$ . AC is a diagonal of the square, and since points A and C both touch the circle, and ABCD is a square, AC is also the diameter of the circle. The radius is equal to half the diameter, so the radius is equal to  $36 \div 2 = 18$ . Thus  $A = \pi(18)^2$ , so the area is equal to  $324\pi$ .
86. *F. Numbers & Operations—Counting Principle.* The only positive integers that leave a remainder of 4 are 8 and 16.
87. *A. Numbers & Operations—Factors, Multiples, Exponents, & Radicals.* We know that  $2^3 = 8$  from prime factorization. We can substitute 8 in for  $2^3$ , but the original expression is  $2^6$ . Instead of multiplying the exponents, we divide them, arriving at  $8^{6 \div 3} = 8^2$ .
88. *F. Numbers & Operations—Numbers.* If the 7,500 mL bucket is only  $\frac{2}{5}$ , then it currently contains  $7,500 \times 2 \div 5 = 3,000$  mL of water. This means there is  $7,500 - 3,000 = 4,500$  mL of water that can be added to fill the bucket. Since there are 1,000 mL in each liter,  $4,500 \div 1,000 = 4.5$  L.
89. *C. Numbers & Operations—Numbers.* Since the number must be in all three sets, it must be odd, so we can eliminate 4 and 6. It must be positive, so we can eliminate 0. Thus, 1.
90. *E. Probability & Statistics—Probability.* If the odds of picking the name of a boy out of a hat is 2 out of 5, then the odds of picking the name of a girl out of a hat is 3 out of 5. So, the ratio of girls to boys is 3:2. We can use a proportion to solve this question:  $\frac{3 \text{ girls}}{2 \text{ boys}} = \frac{15 \text{ girls}}{x \text{ boys}}$ . Cross multiply and solve for  $x$ :  $30 = 2x$ , so  $x = 15$ . But this only tells us the number of boys in the class; the question asks how many more girls there are than boys.  $15 - 10 = 5$ .
91. *B. Algebra—Algebraic Expressions & Equations.* Substitute 6 for  $y$ . This gives us  $6 \div 2 = 3$ . Then,  $2 + 3 = 5$ .
92. *F. Geometry & Measurements—Volume.* Since the surface area of the red colored sides equals 45 square mms, and there are 5 sides that are painted red, then each side is  $45 \text{ square mms} \div 5 \text{ sides} = 9$  square mms. If the area of each side is 9 square mm, then the sides measure 3 mms by 3 mms (the square root of 9). Since the dimensions of the cube measure  $3 \times 3 \times 3$ , the volume is  $3^3 = 27$ .
93. *A. Numbers & Operations—Percents.* Let  $x$  represent the original number. We can represent the fact that  $x$  was decreased by 20% in two different ways:  
 $x - 0.2(x)$ , or simply  $0.8x$ . If from this point, the new number is decreased further by another 20% We repeat the calculations just performed, replacing  $x$  with  $0.8x$  instead:  $0.8x - 0.2(0.8x)$ , or simply  $(0.8)(0.8)(x)$ . Both simplify to  $0.64x$ , which means that compared with the original number  $x$ , the final number is 64% of the original, or 36% less.
94. *F. Numbers & Operations—Scientific Notation.* We need simply to divide the coefficients 4.8 and 1.2 as well as the base and exponents. In this case,  $4.8 \div 1.2 = 4$ . When dividing exponential expressions with the same base, simply subtract the numbers in the exponents. In this case, since the

base is 10 for both expressions, subtract 4 from 6. This gives us  $10^2$ . The resulting expression is  $4 \times 10^2$ .

95. B. *Numbers & Operations—Ratios & Proportions*. Setting up a proportion is the most reliable way to solve this question:  $\frac{7 \text{ oz.}}{3.5 \text{ dollars}} = \frac{16 \text{ oz.}}{x \text{ dollars}}$ . Cross multiply and solve for  $x$ :  $56 = 7x$ , so  $x = 8$ .
96. E. *Algebra—Algebraic Expressions & Equations*. There are 60 minutes in 1 hour. In  $h$  hours, this is  $60h$  minutes. If there are some remainder  $m$  minutes left, then the total number of minutes in  $h$  hours and  $m$  minutes is  $60h + m$ .
97. C. *Algebra—Inequalities*. To find the correct answer, we first need to solve the inequality algebraically. Subtracting 10 from both sides and then dividing both sides by  $-5$ , we get  $c < -1$ . (\*Remember that when you multiply or divide by a negative number, the inequality switches direction!) That means we want an open circle over  $-1$  with the number line shaded to the left.
98. F. *Algebra—Algebraic Expressions & Equations*. We must determine the value of each of the terms in  $xy^2$ . We know that  $3x = 45$ ; solving for  $x$ , we divide both sides by 3. So,  $x = 15$ . We know that  $5y^2$  also equals 45. We divide both sides by 5 and are left with  $y^2 = 9$ . Because  $y^2$  is itself a factor of  $xy^2$ , we don't need to simplify further to solve for  $y$ . Now that we have the values of  $x$  and of  $y^2$ , we simply substitute and multiply:  $(15)(9) = 135$ .
99. A. *Numbers & Operations—Absolute Value*. First, determine the value of  $Q$ .  $|-2| = 2$ , and  $|-8| = 8$ . Therefore,  $Q = (-2 - 8) = -10$ . Substitute  $-10$  in for  $Q$ :  $-|(-10)|$ . After simplifying,  $|10|$  is positive, but the negative on the outside of the absolute value makes it  $-10$ .
100. H. *Probability & Statistics—Averages*. Let  $x$  represent the extra number that is being added to the set. The total of the new set is  $7 \times 12 + x$  because there are 7 numbers with a mean of 12 and we are adding  $x$  to them. This total simplifies to become  $84 + x$ . In order to get the new mean, the total must be divided by 8 because there are 8 numbers including the new one. Therefore, the new mean is  $\frac{(84 + x)}{8}$ . The problem tells us that the new mean is 3 more than the old mean (which was 12) so the new mean is 15. Therefore, we get  $\frac{(84 + x)}{8} = 15$ . To solve this equation, first multiply both sides by 8 and then subtract 84 from both sides. The answer is  $x = 36$ .
101. C. *Geometry & Measurements—Area & Perimeter*. If a bedroom is 10 feet by 12 feet, then the area of that rectangle is  $10 \times 12 = 120$  square feet. Each tile is also a square and is  $4 \times 4 = 16$  square inches. However, working with inches and feet is difficult, so we will convert the tile into square feet. Since there are 12 inches in a foot, the 4-inch tile is  $\frac{1}{3}$  of a foot on each side.  $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$  square foot. To determine how many of these fit into the bedroom, we divide the total square footage of the bedroom by the square footage of a single tile:  $120 \div \frac{1}{9} = 1,080$ . Remember that dividing a number by a fraction is the same as multiplying that number by the reciprocal of the fraction (in this case, 9).
102. F. *Numbers & Operations—Counting Principle*. There are 4 different letters in the word "MATH". Since we are asked for different ways to rearrange the letters, the order of the letters is important. We can write out different possible combinations (MATH, MAHT...etc.), but this would take a long time. Instead, we can tell that keeping one letter in the same (in the above, the letter "M") results in 6 different ways to rearrange the remaining letters (in the above, "A," "T," and "H"). We can repeat this for each of the four letters, meaning there are a total of  $6 \times 4 = 24$  different combinations.
103. A. *Algebra—Algebraic Expressions & Equations*. Find a power of 2 and a power of 3 that multiply together to make 72. We know  $8 \times 9 = 72$ .  $2^3 = 8$  and the power  $3^2 = 9$ . This means that  $x = 3$  and  $y = 2$ . Therefore,  $x + y = 5$ .
104. G. *Numbers & Operations—Numbers*. One way to solve is to choose any  $x$ -value from the table, plug it into each answer choice, and see if you get the correct corresponding  $y$ -value. If we plug 1 in



for  $x$  in each answer choice, all answer choices work except for  $y = x + 5$  and  $y = x + 21$ . If we plug 2 in for  $x$ , only  $y = 3x + 1$  works.

105. B. *Numbers & Operations—Ratios & Proportions*. When two polygons are similar, their corresponding sides are proportional. Here,  $\frac{AB}{BC} = \frac{WX}{XY}$ , or  $\frac{12}{6} = \frac{8}{x}$ . Using cross multiplication,  $12x = 48$ , so  $x = 4$ .
106. E. *Numbers & Operations—Word Problems*.  $\frac{3}{5}$  of a packet of iodine tablets, which is what Kevin knows he must use per day, is 3 iodine tablets, since there are 5 iodine tablets in each packet. Kevin has in total 20 iodine packets  $\times$  5 iodine tablets per packet = 100 iodine tablets. Since Kevin needs 3 iodine tablets, we know that he has enough iodine for 33 days:  $100 \div 3 = 33.33$ . The remainder means that Kevin doesn't have enough iodine for the 34<sup>th</sup> day.
107. B. *Geometry & Measurements—Angles*. The angles opposite the  $45^\circ$  and  $50^\circ$  are called vertical angles. Vertical angles have the same degree measurements. So, the triangle formed by the three-intersecting straight lines has to angles that measure  $45^\circ$  and  $50^\circ$ . Since the sum of the interior angles of a triangle equals  $180^\circ$ , the remaining angle (which is the vertical angle of  $x^\circ$ ) has a measurement of  $180 - 45 - 50 = 85$ . Therefore,  $x = 85^\circ$ .
108. H. *Numbers & Operations—Numbers*. Since the 4<sup>th</sup> number is 68, to find the 5<sup>th</sup> number, first add 2.  $68 + 2 = 70$ . Then, we double this, for  $70(2) = 140$ . To find the 6<sup>th</sup> number, repeat the equation:  $(140 + 2)(2) = 284$ . To find the 7<sup>th</sup> number, do the same:  $(284 + 2)(2) = 572$ .
109. B. *Probability & Statistics—Averages*. The broker earned \$9,000, which was 3% of his total sales. If  $x$  represents his total sales, we can calculate  $x$  using the proportion  $\frac{9,000}{x} = \frac{3}{100}$ . We cross-multiply this to get  $3x = 9,000 \times 100$ . Solving for  $x$ , we get  $x = 300,000$ . This was the broker's total sales over 3 months, so his average sales rate was  $300,000 \div 3 = 100,000$ .
110. F. *Geometry & Measurements—Area & Perimeter*. We know that the formula for perimeter is  $P = 2l + 2w$ . We are told that  $P = 70$  and that  $l = w + 9$ . Plugging those into the equation gives us  $70 = 2(w + 9) + 2w$ , which we can simplify to  $70 = 2w + 18 + 2w$ , or  $4w = 52$ , or  $w = 13$ . That is the width; we're trying to find the length, which is  $13 + 9 = 22$ .
111. B. *Algebra—Algebraic Expressions & Equations*. Looking at the answer choices, we notice that every choice starts with  $a^2$ , so we must distribute the right side of the equation, giving us  $y = a^2 - 5a$ . To find the value of  $y - 1$ , we just subtract 1 from both sides, giving us  $a^2 - 5a - 1$ .
112. F. *Algebra—Algebra in Context*. This problem gives you an equation with several variables, but it also gives you a number to plug in for every variable except one, so just plug in the numbers they've given you:  $600 = 25 \times 50 - (12 \times 50 + b)$ . Then solve for the remaining variable:  $600 = 1,250 - 600 - b$ . This results in  $600 = 650 - b$ , or  $-50 = -b$  which simplifies to  $b = 50$ .
113. D. *Numbers & Operations—Ratios & Proportions*. Any problem involving a scale map can be solved using a proportion. We must always make sure we use matching units so we must convert 2.5 feet to 30 inches, giving us  $\frac{1}{6} = \frac{30}{x}$ , so  $x = 180$ . However, that number is in yards, but the answer choices are all in feet, so we must convert 180 yards to 540 feet.
114. E. *Algebra—Algebra in Context*. A percent is a part of a whole, with the whole being 100. To find  $p$  percent of any value, we must multiply that value by  $p \div 100$ . The value of two \$100,000 houses is \$200,000, so the commission earned will be  $(p \div 100) \times 200,000$ , which simplifies to  $2,000p$ .